§ 2.4 #23

$f = \{ (-3,1), (0,4), (2,0) \}$

$g = \{ (-3,2), (1,2), (2,6), (4,0) \}$ $\quad 2 \notin \mathcal{D}(f \circ g)$

$\Rightarrow f \circ g = f(g(x)) = \{ (-3,0), (1,0), (4,4) \}$

$f(g(-3)) = f(2) = 0 \rightsquigarrow (-3,0)$

$f(g(1)) = f(2) = 0 \rightsquigarrow (1,0)$

$f(g(2)) = f(6) \nexists$

$f(g(4)) = f(0) = 4 \rightsquigarrow (4,4)$

Is f a function? Yes. No reps in x-coord.

Is f 1-to-1? Yes. No reps in y-coord.

Is g a function? Yes. No reps in x-coord

Is g 1-to-1? No. $g(-3) = g(1) = 2$

so $y_1 = y_2 = 2$, but $x_1 = -3 \neq x_2 = 1$.

Prove 1-to-1 : General argument.

Prove Not 1-to-1 : Find one spot where $x_1 \neq x_2$, but $y_1 = y_2$ is all you need.

Prove $f(x) = \dfrac{x+1}{x-2}$ is 1-to-1 :

Pf Suppose $f(x_1) = f(x_2)$. Then

$$\frac{x_1+1}{x_1-2} = \frac{x_2+1}{x_2-2}$$

LCD $= (x_1-2)(x_2-2)$

$$\Rightarrow \left(\frac{x_1+1}{x_1-2}\right)\left(\frac{x_2-2}{x_2-2}\right) = \left(\frac{x_2+1}{x_2-2}\right)\left(\frac{x_1-2}{x_1-2}\right)$$

May safely skip this step when it's an "=" (But not ">" or "<")

$$\Rightarrow (x_1+1)(x_2-2) = (x_2+1)(x_1-2)$$

As long as you get here (cross-multiply works, here)

$$x_1x_2 - 2x_1 + x_2 - 2 = x_2x_1 - 2x_2 + x_1 - 2$$

$$-2x_1 + x_2 = -2x_2 + x_1$$

$$-x_1 \quad -x_2 = \quad -x_2 \quad -x_1$$

$$-3x_1 = -3x_2$$

$$\frac{-3x_1}{-3} = \frac{-3x_2}{-3}$$

may skip. Don't clutter the line above with your division by -3, the Need to see the $-3x_1 = -3x_2$ plain.

$$x_1 = x_2$$

$\Rightarrow$ 1-to-1 ▨

At Speed :

S $f(x_1) = f(x_2)$. Then $\dfrac{x_1+1}{x_1-2} = \dfrac{x_2+1}{x_2-2}$

$$(x_1+1)(x_2-2) = (x_2+1)(x_1-2)$$

$$\cancel{x_1x_2} - 2x_1 + x_2 \cancel{-2} = \cancel{x_2x_1} - 2x_2 + x_1 \cancel{-2}$$

$$-2x_1 + x_2 = -2x_2 + x_1$$

$$-3x_1 = -3x_2$$

$$x_1 = x_2 \quad ▨$$