$f+g,\ f-g,\ f\cdot g = \boxed{fg}$ ARITHMETIC

$f \circ g = f(g(x))$

composed with

$f(x) = x^2+2,\ g(x) = \sqrt{3x-1}$ $\Longrightarrow$ Intersect

$f \pm g = x^2+2 \pm \sqrt{3x-1}$

$f \cdot g = (x^2+2)(\sqrt{3x-1})$

$\mathcal{D} = \mathcal{D}(f) \cap \mathcal{D}(g)$

AND

$\frac{f}{g} = \frac{x^2+2}{\sqrt{3x-1}}$

Conjunction Restriction

$\mathcal{D}(f \pm g) = \mathcal{D}(f \cdot g) = \mathcal{D}(f) \cap \mathcal{D}(g)$

$= \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g)\}$

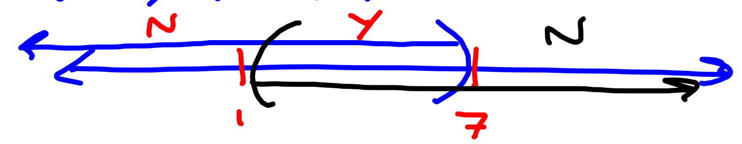
$$\mathcal{D}\left(\frac{f}{g}\right) = \mathcal{D}(f) \cap \mathcal{D}(g) \cap \{x \mid g(x) \neq 0\}$$

f has to have x in its diet.

g has to have x in its diet.

Some intersections:

$(-\infty, 7) \cap (1, \infty)$

$=$ 

AND

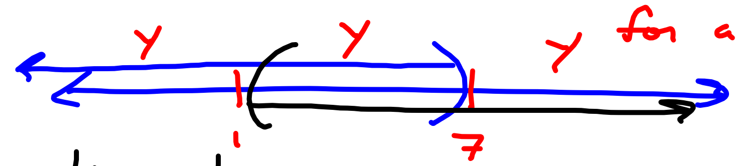
Need the black AND the blues

$= (1, 7)$

By contrast, union:

$(-\infty, 7) \cup (1, \infty)$

We're focused on the "AND" for a while.

$=$ 

OR

In mathematics, the "OR" is not exclusive. It's inclusive, so the $(-\infty, 7) \cap (1, \infty)$ part is included.

Let's talk about $\mathcal{D}\left(\frac{f}{g}\right)$ $\qquad \frac{f}{g} = \frac{x^2+2}{\sqrt{3x-1}}$

The quotient <u>also</u> needs to keep $\sqrt{3x-1} \neq 0$

Domain

$f(x) = x^2+2$. $\mathcal{D}(f) =$

Hint: The real numbers are closed under addition and multiplication.

$x \in \mathbb{R}$ and $y \in \mathbb{R}$

$\Rightarrow$ $x+y \in \mathbb{R}$ and $xy \in \mathbb{R}$

So $\boxed{\mathcal{D}(f) = \mathbb{R} = (-\infty, \infty)}$

In general, polynomials are defined (real) $\underbrace{\text{for every } x \in \mathbb{R}}_{\forall x \in \mathbb{R}}$.

Feed f a real #, then f(x) is a real number. (Keep it real.)

What about $g(x)$?

$\sqrt{3x-1}$

<u>Need</u> $3x-1$ nonnegative

$3x-1 \geq 0$

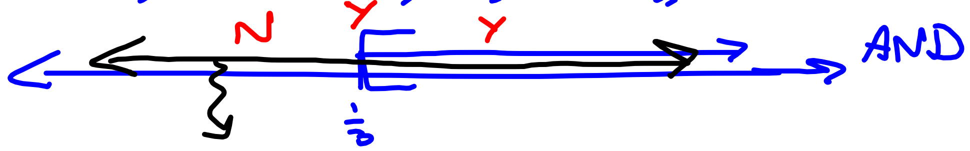
$3x \geq 1$

$x \geq \frac{1}{3}$: $x \in [\frac{1}{3}, \infty)$

$\mathcal{D}(f \pm g) = \mathcal{D}(f \cdot g) = \mathcal{D}(f) \cap \mathcal{D}(g)$

$= (-\infty, \infty) \cap [\frac{1}{3}, \infty) = [\frac{1}{3}, \infty)$

$=$



$= [\frac{1}{3}, \infty)$

check boundary

What about $\frac{f}{g}$?

Same, but with an additional $g(x) \neq 0$ condition.

$g(x) = 0$

$\Rightarrow \sqrt{3x-1} \neq 0$

$\Rightarrow 3x-1 \neq 0$

$\Rightarrow 3x \neq 1$

$x \neq \frac{1}{3}$ So, throw out $x = \frac{1}{3}$

from $\mathcal{D}(f \pm g) = \mathcal{D}(f \cdot g) = [\frac{1}{3}, \infty)$

So $\boxed{\mathcal{D}\left(\frac{f}{g}\right) = \left(\frac{1}{3}, \infty\right)}$

This concludes arithmetic operations.
arithMETic

Domain, in general:

1. $\sqrt{\text{Negative}}$ Bad

$\sqrt{\text{Radicand}}$ Need Radicand ≥ 0

$\sqrt{\text{STUFF}}$ Need $\boxed{\text{STUFF} \geq 0}$

2. $\dfrac{\text{Junk}}{\text{STUFF}}$: $\dfrac{\text{Junk}}{0}$ is bad.

Need $\boxed{\text{STUFF} \neq 0}$

Everything else is good!
$D = (-\infty, \infty)$, always, except for 1 & 2