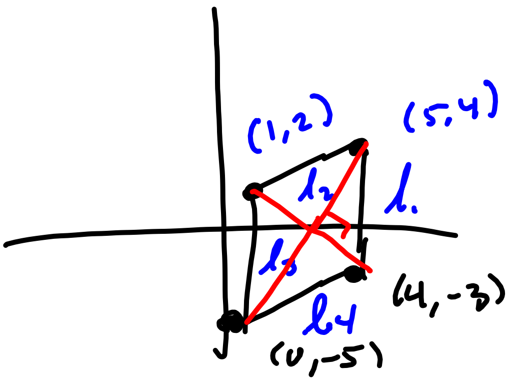
1.4.89

Decide whether the statement is true or false.

The points (0, − 5), (1,2), (4, − 3) and (5,4) form the vertices of a parallelogram.



$\boxed{M1}$ Show slopes of opposite sides are the same

$l_1: \frac{y_2-y_1}{x_2-x_1} = \frac{4-(-3)}{5-4} = \frac{7}{1} = m$

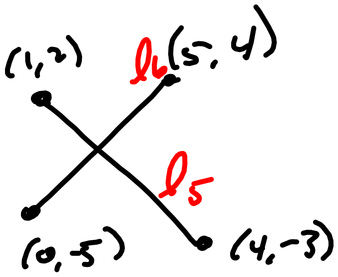
$l_3: m = \frac{2-(-5)}{1-0} = \frac{7}{1} = m$

$l_2: m = \frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2} = m$

$l_4: m = \frac{-3-(-5)}{4-0} = \frac{+2}{4} = \frac{1}{2} = m$

Yawp!

$\boxed{M2}$ Diagonals $\perp$?



$l_5: \frac{-3-2}{4-1} = -\frac{5}{3} = -\frac{5}{3}$

$l_6: \frac{4-(-5)}{5-0} = \frac{4+5}{5} = \frac{9}{5}$

Teacher's full of crap.

Not $\perp$. I guess I was wrong about the diagonals

1 & -1 are inverses (with respect to addition)

$1 + -1 = 0$ = additive identity

2 & $\frac{1}{2}$ are inverses w.r.t. multiplication

$2 \cdot \frac{1}{2} = 1$ = multiplicative identity

$0 + x = x$

$1 \cdot x = x$

$\ell_1 \perp \ell_2 \Longrightarrow m_2 = -\frac{1}{m_1}$

m_2 is the negative reciprocal of m_1.

m_2 is the .. inverse (wrt mult.) of m_1.

1 for what's-his-name.

We were looking for $m_2 = \frac{5}{3}$

$(m_1 = -\frac{3}{5})$

Quadratic Formula

If $ax^2+bx+c=0$, then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

ALWAYS do b^2-4ac first.

Then simplify $\sqrt{b^2-4ac}$.

Then plug in.

Kalista

$x^2-x-6=0$

$a=1$, $b=-1$, $c=-6$

Devin, again. Very irritating.

$b^2-4ac = (-1)^2-4(1)(-6) =$

$= 1+24 = 25 \to \sqrt{25}$

5 | 25
5

$= \sqrt{5\cdot 5} = 5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-1)\pm 5}{2(1)}=\frac{1\pm 5}{2}=\begin{cases}\frac{6}{2}=3\\ \frac{-4}{2}=-2\end{cases}$$

$$\boxed{x\in\{-2,3\}}$$

FACTORING!

$\cancel{1}x^2 - x - 6$

$(1)(-6) = -6$ MAGIC!

$-1 = -2 + 1$ $\quad -2$

$-1x$ $\quad = -3 + 2$

$\underline{-6}$ Sweet!

The magic #!

$x^2 - 3x + 2x - 6$ $\quad$ Factor by Grouping

$= x\left(\frac{x^2}{x} - \frac{3x}{x}\right)$

$= x(x-3) + 2(x-3)$

$x ☺ + 2 ☺$

$(1+2x)^{-\frac{5}{2}}$

$= ☺ (x+2)$

$AB = 0 \longrightarrow$

$= (x-3)(x+2) = 0$

$A = 0$ or $B = 0$

$\Rightarrow$ OPTIONAL $\begin{cases} x - 3 = 0 \text{ or } x + 2 = 0 \\ x = 3 \text{ or } x = -2 \end{cases}$

$\boxed{x \in \{-2, 3\}.}$

The Factor Theorem

If $P(x)$ is a polynomial and $P(c)=0$, then $(x-c)$ is a factor.

Recall $x^2-x-6 = (x-3)(x+2) = P(x)$

$x=3$ is a zero of $P(x)$

$x-3$ is a factor, etc.

$P(c)=0 \iff x-c$ is a factor.

$P(3)=0 \iff x-c$ " " "

$\iff$ means "if and only if" which means "necessary and sufficient," means you can reason from one to the other and back again.

So far, you've ONLY seen: $x-3$ is a factor $\Rightarrow x=3$ is a zero.

SLEDGEHAMMER

$x=3$ is a zero $\Rightarrow x-3$ is a factor.

$56x^2 - 78x - 495$ Use quadratic formula to find the zeros. (or technology).

$P := x \to 56x^2 - 78x - 495$

$P := x \mapsto 56x^2 - 78x - 495$

$solve(P(x) = 0)$

$\frac{15}{4}, -\frac{33}{14}$

This says $P(x) = 56(x - \frac{15}{4})(x + \frac{33}{14})$

$= 56x^2 +$ smaller stuff

Now, trick the teacher

$= 4 \cdot 14 (x - \frac{15}{4})(x + \frac{33}{14})$

$= 4(x - \frac{15}{4})(14)(x + \frac{33}{14})$

$\boxed{= (4x - 15)(14x + 33)}$

$3 \cdot 2 = 2 \cdot 3$ commutative property

(Reals are an abelian group under multiplication)

$P(x) = x^2 - 5x - 3 = 0$

$a = 1,\ b = -5,\ c = -3$

$b^2 - 4ac = (-5)^2 - 4(1)(-3) = 25 + 12 = 37$

$x = \dfrac{5 \pm \sqrt{37}}{2(1)}$ $\Rightarrow$

$P(x) = \left(x - \left(\dfrac{5+\sqrt{37}}{2}\right)\right)\left(x - \left(\dfrac{5-\sqrt{37}}{2}\right)\right)$

$e(P(x) = 0\ )$

$\dfrac{15}{4}, -\dfrac{33}{14}$

$md\left(\left(x - \dfrac{(5+\mathrm{sqrt}(37))}{2}\right) \cdot \left(x - \dfrac{(5-\mathrm{sqrt}(37))}{2}\right)\right)$

$= x^2 - 5x - 3$

SWEET!

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)^2 = a^2 + 2ab + b^2$

Expanding the square of a binomial.

$(x-3)^2 = x^2 - 6x + 9$

$(x+7)^2 = x^2 + 14x + 49$

$(x-13)^2 = x^2 - 26x + 169$

$x^2 - 10x + 7 = (x-5)^2 + \text{something}$

$x^2 - 10x + 5^2 - 25 + 7$

$\frac{10}{2} = 5 \rightsquigarrow 5^2 = 25$

$= (x^2 - 10x + 5^2) - 18$

$= (x-5)^2 - 18$ SET 0

$+18 = 18$

$\Longrightarrow (x-5)^2 = 18$

$\left.\begin{aligned} \sqrt{(x-5)^2} &= \sqrt{18} \\ |x-5| &= \sqrt{18} \end{aligned}\right\}$

whence cometh the "$\pm$" that's so mysterious to high school teachers.

$\Longrightarrow x - 5 = \pm\sqrt{18}$

$\Longrightarrow x = 5 \pm \sqrt{18}$

$\boxed{= 5 \pm 3\sqrt{2}}$

Square Root Property:

$A^2 = B$

$\left\{\begin{aligned} \sqrt{A^2} &= \sqrt{B} \\ |A| &= \sqrt{B} \end{aligned}\right.$

$\Longrightarrow A = \pm\sqrt{B}$

$$\begin{array}{r|l} 2 & 18 \\ 3 & 9 \\ & 3 \end{array}$$

$\sqrt{18} = 3\sqrt{2}$

$3\sqrt{2} = \sqrt{3^2}\sqrt{2} = \sqrt{3^2 \cdot 2} = \sqrt{18}$

$\sqrt{}$ means PRINCIPAL SQUARE ROOT