# Amt pure alcohol = Amt Pure alcohol

10% & 15% mixed to give 12% mix

| | Vol | Pure Alcohol | gals. |
|---|---|---|---|
| 10% | x | .1x | |
| 15% | y | .15y | |
| 12% | 20 | .12(20) | |
| | x+y | | |

10% mix $\frac{\text{.1 gal pure alcohol}}{\text{1 gal 10\% mix}}$

10% .1x means

$\left(\frac{\text{.1 gal alc.}}{\text{1 gal 10\% mix}}\right)$(x gal 10% mix)

Pure Alc. $.1x + .15y = .12(20)$

Total Vol. $x + y = 20$ Auxiliary Eq'n.

Eliminate y in 1st eq'n:

$y = 20 - x$

$.1x + .15y = .12(20)$

Lexicon Let x = # of gallons of 10% alcohol

y = " " " " 15% alcohol.

$.1x + .15(20 - x) = .12(20)$

$.1x + .15(20) - .15x = .12(20) = 2.4$

$-.05x + .15(20) = 2.4$

$$-.05x = 2.4 - .15(20)$$

$$x = \frac{2.4 - .15(20)}{-.05} = \frac{2.4-3}{-.05}$$

$$= \frac{-.6}{-.05} = \frac{-\frac{6}{10}}{-\frac{5}{100}} = +\frac{6}{10}\cdot\frac{100}{5} = \frac{6(10)}{5} = 12\ gal = x$$

$$\Rightarrow y = 20 - x = 20 - 12 = 8\ gal = y$$

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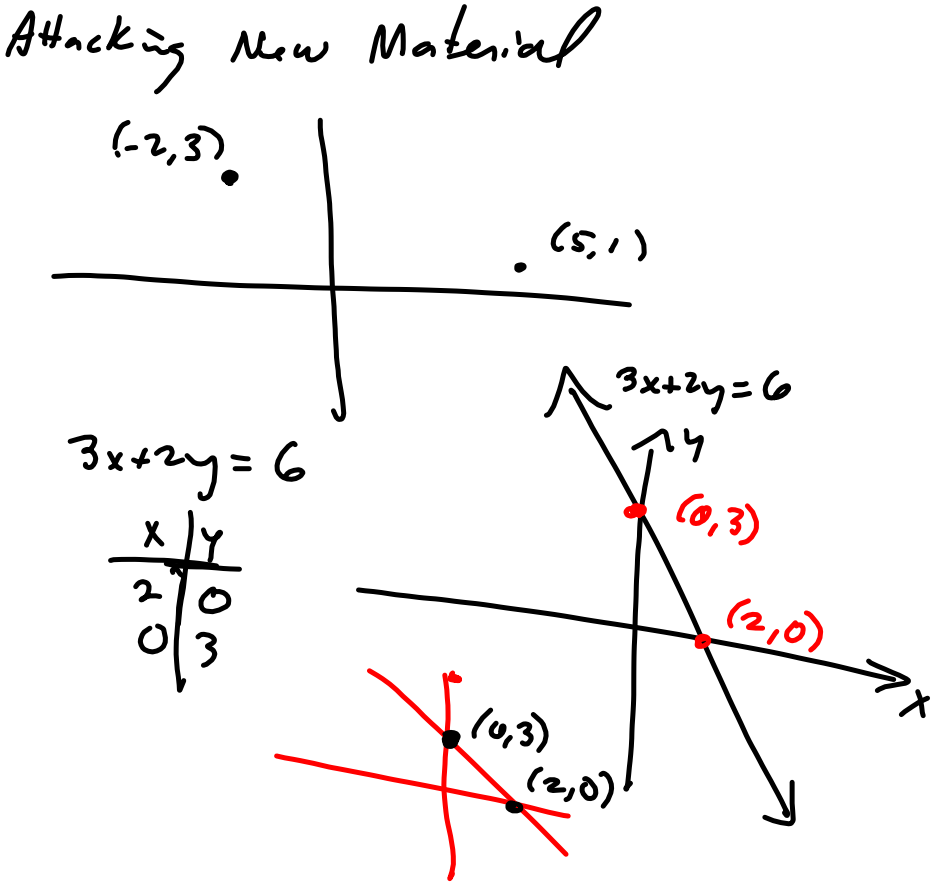
gave 12 gal of 10%, unknown final volume

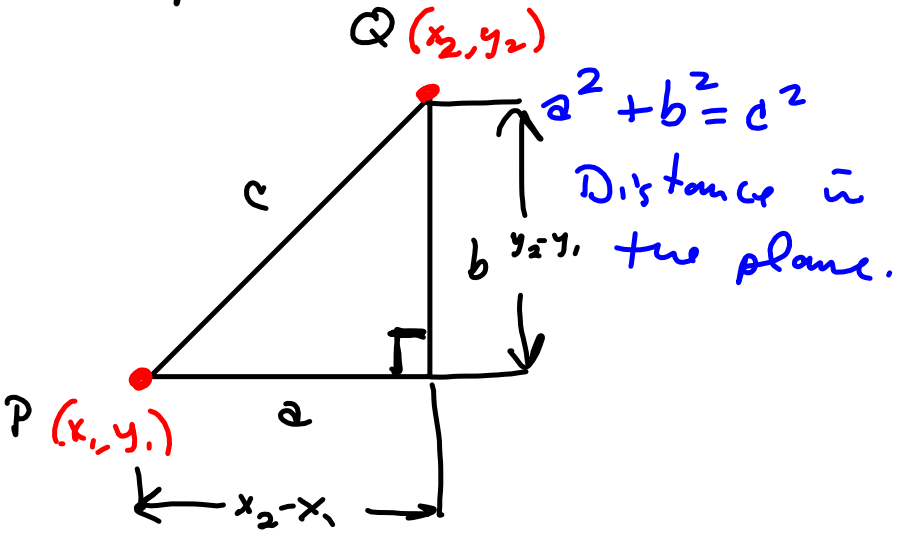
| | vol | Alcohol |
|---|---|---|
| 10% | 12 | .1(12) |
| 15% | y | .15y |
| 12% | 12+y | .1(12)+.15y |

$$.1(12) + .15y = .1(12) + .15y$$

Attacking New Material



§1.3 Pythagorean Theorem & Distance & Midpoints.




Distance from P to Q is D(P, Q)

$c^2 = a^2 + b^2$

$D(P,Q)^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$\sqrt{D(P,Q)^2} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$|D(P,Q)| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Distance in the plane is positive

$\sqrt{3^2} = 3$ $\qquad \sqrt{x^2} = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$\sqrt{(-3)^2} = 3$ $\qquad = |x|$

$|3| = 3$

$|-3| = 3 = -(-3)$

$$|D(P,Q)| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$\Rightarrow D(P,Q) = \pm\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

By convention, $D(P,Q)$ is positive!

$$\rightarrow \boxed{D(P,Q) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$$

$(x-5)^2 = 16$

$\sqrt{\quad} = \sqrt{\quad}$

$|x-5| = \sqrt{16} = 4$

$x-5 = \pm 4$

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Midpoint of $(x_1, y_1)$ & $(x_2, y_2)$ is

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$(1,2), (-3, \pi) = \left(\frac{1-3}{2}, \frac{2+\pi}{2}\right) = \left(-1, \frac{2+\pi}{2}\right)$

POIFECK

$= \left(-1, \frac{2(1+\frac{\pi}{2})}{2}\right) = (-1, 1+\frac{\pi}{2})$

A circle is the locus of points equidistant from a fixed point called "the center."

Let $(h,k)$ = center

$r$ = the radius.

Let $(x,y)$ be a point on the circle. Its distance to $(h,k)$ is $r$.

$$\sqrt{(x-h)^2+(y-k)^2} = r$$

$$(\ )^2 = (\ )^2$$

$$\boxed{(x-h)^2+(y-k)^2 = r^2}$$

Standard form of a circle.

$$\sqrt{x^2} = |x|$$

$$(\sqrt{x})^2 = x \quad (\text{to be real, } x \geq 0 \text{ to start with!})$$

Write $x^2+y^2 = 4x+3y$

$-4x-3y \quad = -4x-3y$

$x^2-4x \quad +y^2-3y \quad = 0$

$(x+3)^2 = (x+3)(x+3) = x^2+3x+3x+9$

$= x^2+2(3x)+9$

$(x-5)^2 = x^2-5x-5x+25$

$= x^2-10x+25$

$= x^2-2(5x)+25$

$(x+b)^2 = x^2+2bx+b^2$ → Find Me!

$x^2-4x+$

$= x^2-2(b)(x)$

$b=2$

$x^2-4x =$

$= x^2-2(2x)+2^2-2^2$

$=(x-b)^2-4$

$= (x-2)^2-4$

?

$x^2+10x+5^2$

$\frac{10}{2} = 5 \rightsquigarrow 5^2$

$x^2-4x+2^2+y^2-3y+\left(\frac{3}{2}\right)^2 = 0+4+\frac{9}{4} = \frac{16+9}{4} = \frac{25}{4}$

$\frac{-4}{2} = -2 \rightarrow (-2)^2 = 4$

$\frac{3}{2} \rightarrow \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$

$(x-2)^2+\left(y-\frac{3}{2}\right)^2 = \frac{25}{4}$

$(h,k) = \left(2,\frac{3}{2}\right)$

$r = \sqrt{\frac{25}{4}} = \frac{5}{2} = r$

58 $x^2+10x+y^2-8y = -40$

$x^2+10x+5^2+y^2-8y+4^2 = -40+25+16$

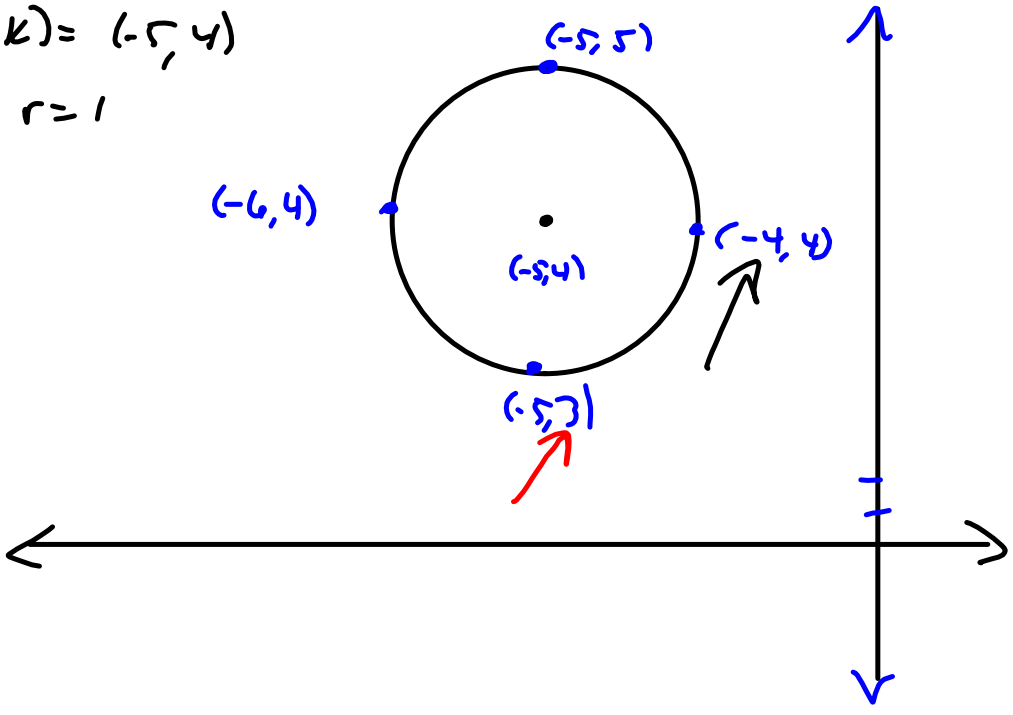
$\frac{10}{2} = 5 \rightarrow 5^2$

$\frac{8}{2} = 4 \rightarrow 4^2$

$(x+5)^2+(y-4)^2 = 1$

$(h,k) = (-5,4)$

$r = 1$



$10x^2 - 37x - 182$

$-1820$

$-37 = -38 + 1 \qquad -38$

$= -27 + 10 \qquad -270$

$= -17 + 20 \qquad -340$

$b^2 - 4ac = (-37)^2 - 4(10)(-182)$

$= 1369 + 7280$

$= 7416$

$x = \frac{37 \pm 6\sqrt{206}}{2(10)} = x = \frac{37 \pm 6\sqrt{206}}{20}$

$10\left(x - \left(\frac{37+6\sqrt{206}}{20}\right)\right)\left(x - \frac{37-6\sqrt{206}}{20}\right)$