

Remember to use separate paper for everything except your name. Leave a margin in the top left corner. Spread out your work. Use only one column for your work. *Submit problems in order!!!*

1. (10 pts) Form a polynomial of *minimal degree* in *factored form* that has real coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

$$x = 1, \text{ multiplicity } 3; \quad x = 3 + i, \text{ multiplicity } 2; \quad x = 2, \text{ multiplicity } 1.$$

2. (10 pts) Use synthetic division to find $P(3)$ if $P(x) = 7x^5 - 2x^4 + 11x^3 + x^2 - 173x - 4$

3. (5 pts) Represent the work you just did on the previous problem by writing $P(x)$ in the form $Dividend = Divisor \bullet Quotient + Remainder$.

4. Suppose $f(x) = (x+2)^2(x-1)^3(x-7)(x+5) = x^7 - x^6 - 42x^5 - 26x^4 + 185x^3 + 15x^2 - 272x + 140$. I'm showing you both factored and expanded form to help you answer the following:

- a. Solve the inequality $f(x) \leq 0$. Your sign pattern for this one will be helpful in the next two. You just have to interpret what you're seeing.
- b. (10 pts) Provide a rough sketch of f , using its zeros, their respective multiplicities and its end behavior. Include x - and y -intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better quality graph.

- c. (5 pts) What is the domain of $\sqrt{\frac{(x+2)^2(x-7)}{(x-1)^3(x+5)}}$?

5. Let $f(x) = 4x^5 + 16x^4 + 25x^3 + 23x^2 - 16x - 52$.

- a. (5 pts) Use Descartes' Rule of Signs to determine the possible number of positive and negative zeros of f .
- b. (5 pts) List all possible rational zeros of f .
- c. (Bonus 5 pts) Find the smallest possible integer bounds on positive and negative zeros.

6. (10 pts) Find the *real* zeros of $f(x) = 4x^5 + 16x^4 + 25x^3 + 23x^2 - 16x - 52$. Then factor f over the set of **real numbers**. This should involve an irreducible quadratic factor.

(If things go haywire, come up with a *plausible*-looking polynomial, in factored form, with the right number of real roots and 2 nonreal roots. The 2 nonreal roots will still be living inside the irreducible quadratic factor, so you'll have to make up a quadratic factor with nonreal zeros).

7. (5 pts) Find the remaining (nonreal) zeros of f and factor f over the set of **complex numbers**. This step requires breaking down the quadratic piece that's irreducible over the real numbers. The fundamental theorem tells us that *nothing* is irreducible over the complex numbers.

(You can still get full credit for this one, even if things went haywire, in #6, if you solve the depressed equation, correctly, and display your *plausible*-looking follow-up to your *plausible*-looking answer to #6. The more you know about what you're pointing towards, the more points you'll earn.)

8. (5 pts) You don't need to graph $R(x) = \frac{3x^3 - 14x^2 + 23x - 10}{x^2 - x - 2}$, here, but I do want to see its asymptotes.

Hint: This function has no holes.

9. (10 pts) Sketch the graph of $F(x) = \frac{3x^2 - 14x + 15}{x^2 - 5x - 14}$. Show all asymptotes and intercepts.

ANSWER ANY TWO (2) OF THE FOLLOWING, FOR UP TO 20 BONUS POINTS!!!

- B1** (10 pts) Form a polynomial of *minimal degree* in *factored form* that has **rational** coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong.



Zeros: $x = 2 + \sqrt{3}$, multiplicity 1;
 $x = 2 + 7i$, multiplicity 2;
 $x = -5$, multiplicity 17.

- B2** Solve both of the following absolute value inequalities.

a. (5 pts) $|2x - 5| + 5 \geq 7$

b. (5 pts) $|2x - 5| + 7 < 5$

- B3** (10 pts) Sketch the graph of $R(x) = \frac{3x^3 - 14x^2 + 23x - 10}{x^2 - x - 2}$. Hints:

a. You already found $R(x)$'s asymptotes.

b. One of $R(x)$'s x -intercepts is $\left(\frac{2}{3}, 0\right)$.

- B4** (10 pts) Sketch the graph of $G(x) = \frac{3x^3 - 5x^2 - 27x + 45}{x^3 - 2x^2 - 29x - 42}$. Hint: $G(x)$ looks exactly like $F(x)$, from #9, except it has a hole.

- B5** John takes 5 hours to do a job it takes Jill 8 hours to do. If John shows up for the job 2 hours late, and he works with Jill until the job is done, how many hours does Jill work? How many hours does John work?