121 E2 FALL, 2017

① 5 pts KINDNESS

② $R = \{(5,3), (2,4), (7,4), (4,7)\}$

a) Yes

b) $\mathcal{D} = \{5, 2, 7, 4\}$

c) $\mathcal{R} = \{3, 4, 7\}$

d) (2,4) & (7,4) both have same y-coordinate, but different x-coord.
1-to-1 means if $y_1 = y_2$, then $x_1 = x_2$,
but we have $y_1 = y_2$ and $x_1 \neq x_2$.

③ $f(x) = \sqrt{x+11}$, $g(x) = x^2 + 2x - 35$

a) $\mathcal{D}(f) = \{x \mid x + 11 \geq 0\} = \{x \mid x \geq -11\}$

$= [-11, \infty)$

b) $\mathcal{D}(g) = \mathbb{R} = (-\infty, \infty)$

c) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+11}}{x^2+2x-35}$

~~d) $\mathcal{D}\left(\frac{f}{g}\right) = \mathcal{D}(f) \cap \mathcal{D}(g)$~~

121 F2

(3d) $\mathcal{D}(f/g) = \mathcal{D}(f) \cap \mathcal{D}(g) \cap \{x \mid g(x) \neq 0\}$

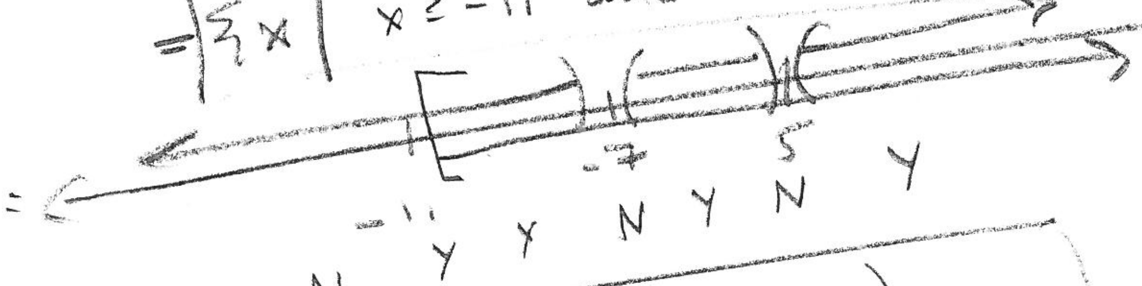
OR $\{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \text{ and } g(x) \neq 0\}$

$= \{x \mid x \geq -11 \text{ and } x^2+2x-35 \neq 0\}$

Scratch: $x^2+2x-35 = (x+7)(x-5) = 0$

$\Rightarrow x \in \{-7, 5\}$ is NOT in $\mathcal{D}$.

$= \boxed{\{x \mid x \geq -11 \text{ and } x \neq -7, 5\}}$ AND

=


$= \boxed{[-11,-7) \cup (-7,5) \cup (5,\infty)}$

(e) $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)+11} =$

$= \boxed{\sqrt{x^2+2x-35+11}}$

121 E2

(f) $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

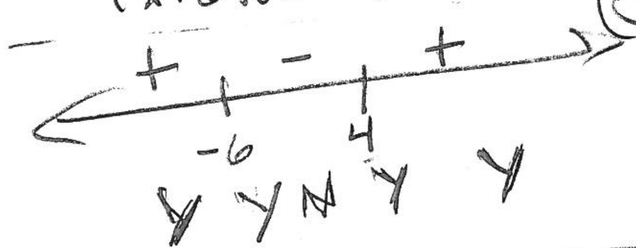
$= \{x \mid g(x) \in \mathcal{D}(f)\} = \{x \mid x^2 + 2x - 35 \geq -11\}$

Scratch:

$x^2 + 2x - 35 \geq -11$

$x^2 + 2x - 24 \geq 0$

$(x+6)(x-4) \geq 0$ ≥ 0



$= \boxed{(-\infty, -6] \cup [4, \infty) = \mathcal{D}(f \circ g)}$

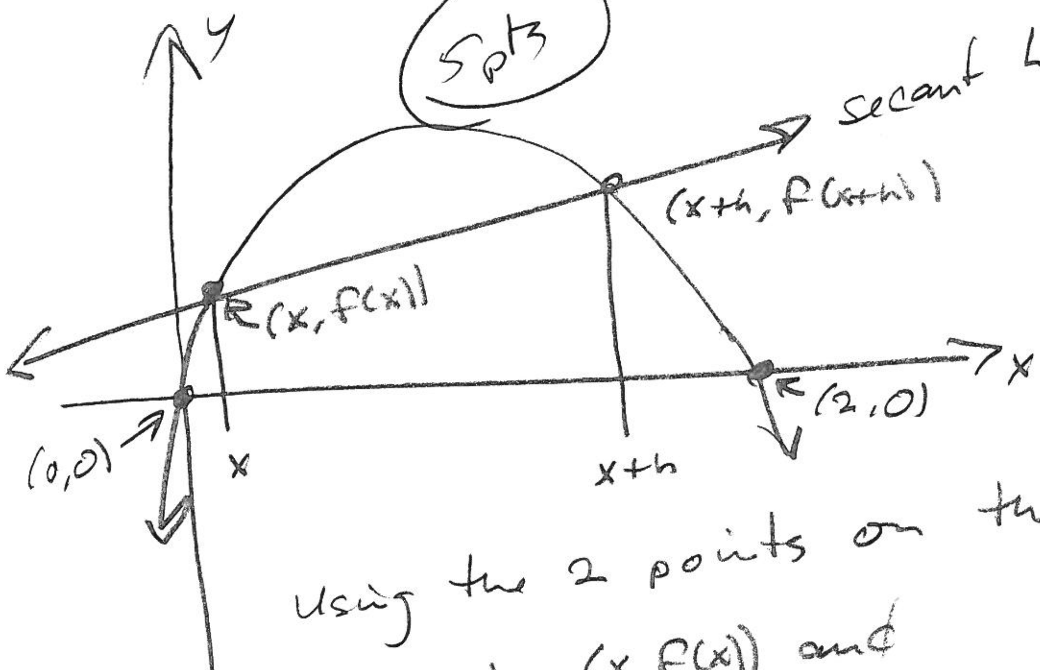
$\boxed{= \{x \mid x \leq -6 \text{ OR } x \geq 2\}}$

121 E2

(4) $f(x) = -x^2+2x \longrightarrow$

$$\frac{f(x+h)-f(x)}{h} = \frac{-(x+h)^2+2(x+h)-[-x^2+2x]}{h}$$

5pts



Using the 2 points on the secant line $(x_1,y_1) = (x, f(x))$ and $(x_2,y_2) = (x+h, f(x+h))$, we see that the slope of the secant line is

$$\frac{y_2-y_1}{x_2-x_1} = \frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

<u>IS</u> the difference quotient.

121 E2

(5) From #4:

$$= \frac{-(x^2+2xh+h^2)+2x+2h+x^2-2x}{h}$$

$$= \frac{-x^2-2xh-h^2+2x+2h+x^2-2x}{h}$$

5pts

$$= \frac{-2xh-h^2+2h}{h}$$

$$= \frac{h(-2x-h+2)}{h} = \boxed{-2x-h+2}$$

Bonus #1:

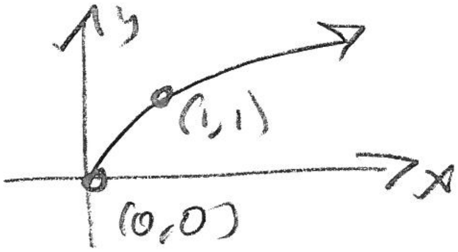
$$\boxed{h \to 0 \quad \longrightarrow \quad -2x+2 = f'(x)}$$

5pts

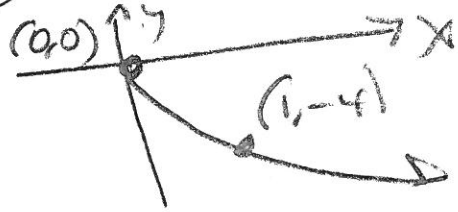
121 E2

(6) 20pts $g(x) = -4\sqrt{2x-18}+5$

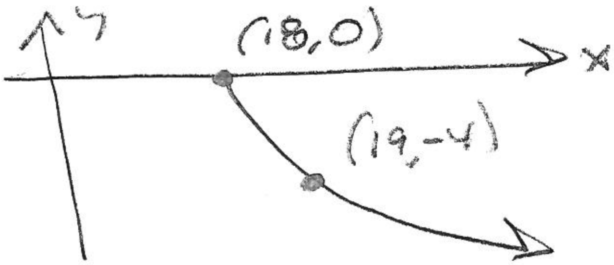
(0) $f(x) = \sqrt[6]{x}$



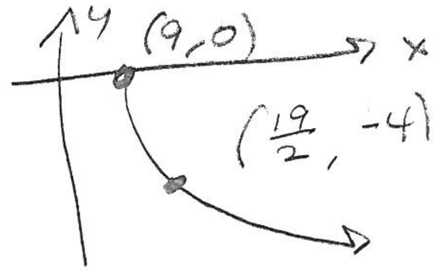
(1) $-4f(x) = -4\sqrt[6]{x}$



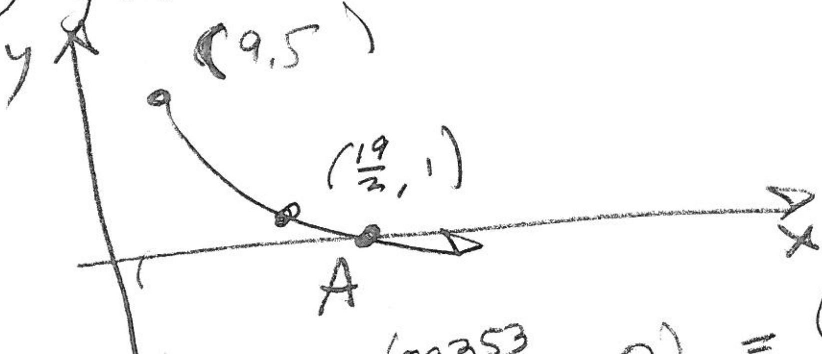
(2) $-4f(x-18) = -4\sqrt[6]{x-18}$



(3) $-4f(2x-18)$
$= -4\sqrt[6]{2x-18}$



(4) $g(x) = \#3 + 5$



$A = \left(\frac{89353}{8192}, 0\right) = \left(\frac{\left(\frac{5}{4}\right)^6+18}{2}, 0\right)$

$\approx (10.90734863, 0)$

121 E2

(b) $\mathcal{D}(g) = [9, \infty)$

$\mathcal{R}(g) = (-\infty, 5]$

(c) y-int : $\nexists$ $x = 0 \notin \mathcal{D}$.

x-int:

$-4\sqrt[6]{2x-18} + 5 = 0$

$-4\sqrt[6]{2x-18} = -5$

$\sqrt[6]{2x-18} = \frac{5}{4}$

$2x - 18 = \left(\frac{5}{4}\right)^6$

$2x = \left(\frac{5}{4}\right)^6 + 18$

$x = \frac{\left(\frac{5}{4}\right)^6 + 18}{2}$

$$\boxed{A = \left(\frac{\left(\frac{5}{4}\right)^6 + 18}{2}, 0\right) \approx (10.90734863, 0)}$$

$= \left(\frac{89353}{8192}, 0\right)$

121 E2

(7) $f(x) = \frac{1}{x+3}$ is 1-to-1

$\boxed{\text{Pf}}$ $\mathcal{S}$ $f(x_1) = f(x_2)$. Then

$$\frac{1}{x_1+3} = \frac{1}{x_2+3}$$

$$\Longrightarrow x_2+3 = x_1+3$$

$$\Longrightarrow x_2 = x_1 \quad \blacksquare$$

(8) $\boxed{T = k\frac{xw}{\sqrt[3]{z}}}$

(B1) $-2x-h+2 \xrightarrow{h \to 0} \boxed{-2x+2 = f'(x)}$

(B2) $$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{3(x+h)} - \sqrt{3x}}{h}$$

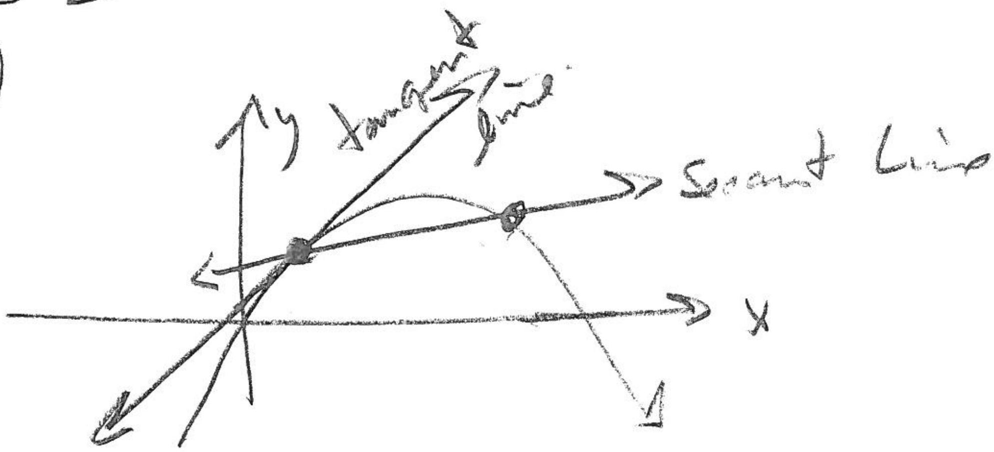
$$= \left(\frac{\sqrt{3(x+h)} - \sqrt{3x}}{h}\right)\left(\frac{\sqrt{3(x+h)} + \sqrt{3x}}{\sqrt{3(x+h)} + \sqrt{3x}}\right)$$

$$= \frac{3x+3h-3x}{h(\sqrt{3(x+h)} + \sqrt{3x})} = \frac{3\cancel{h}}{(\sqrt{3x+3h} + \sqrt{3x})\cancel{h}} \xrightarrow{h \to 0} \boxed{\frac{3}{2\sqrt{3x}}}$$

$$= \frac{1}{\sqrt{3x+3h} + \sqrt{3x}}$$

121 E2

B3



B4 $4x^2 - 7x + 11$

$$= 4\left(x^2 - \frac{7}{4}x + \left(\frac{7}{8}\right)^2\right) + 11 - 4\left(\frac{7}{8}\right)^2$$

Scratch: $11 - 4\left(\frac{49}{64}\right)$

$$= 11 - \frac{49}{16} = \frac{176 - 49}{16} = \frac{127}{16}$$

$$\boxed{= 4\left(x - \frac{7}{8}\right)^2 + \frac{127}{16}} \quad \boxed{(h,k) = \left(\frac{7}{8}, \frac{127}{16}\right)}$$

$$= (.875, 7.9375)$$

121

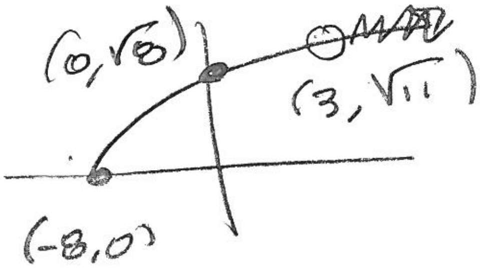
B5

E2

$$h(x) = \begin{cases} \sqrt{x+8} & \text{if } x<3 \\ x^2-5x+4 & \text{if } x \ge 3 \end{cases}$$

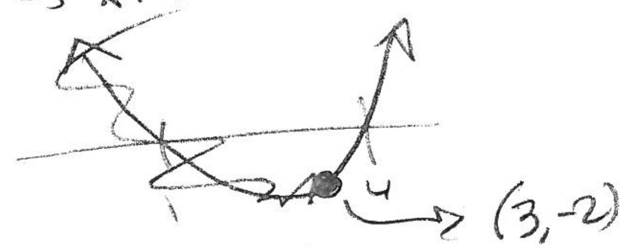
$x<3$

$\sqrt{x+8}$



$x \ge 3$

$x^2-5x+4 = (x-1)(x-4)$



$\sqrt{8}$

$= \sqrt{2^2 \cdot 2}$

$= 2\sqrt{2}$

$(3-1)(3-4)$

$= (2)(-1) = -2$

COMBINE :