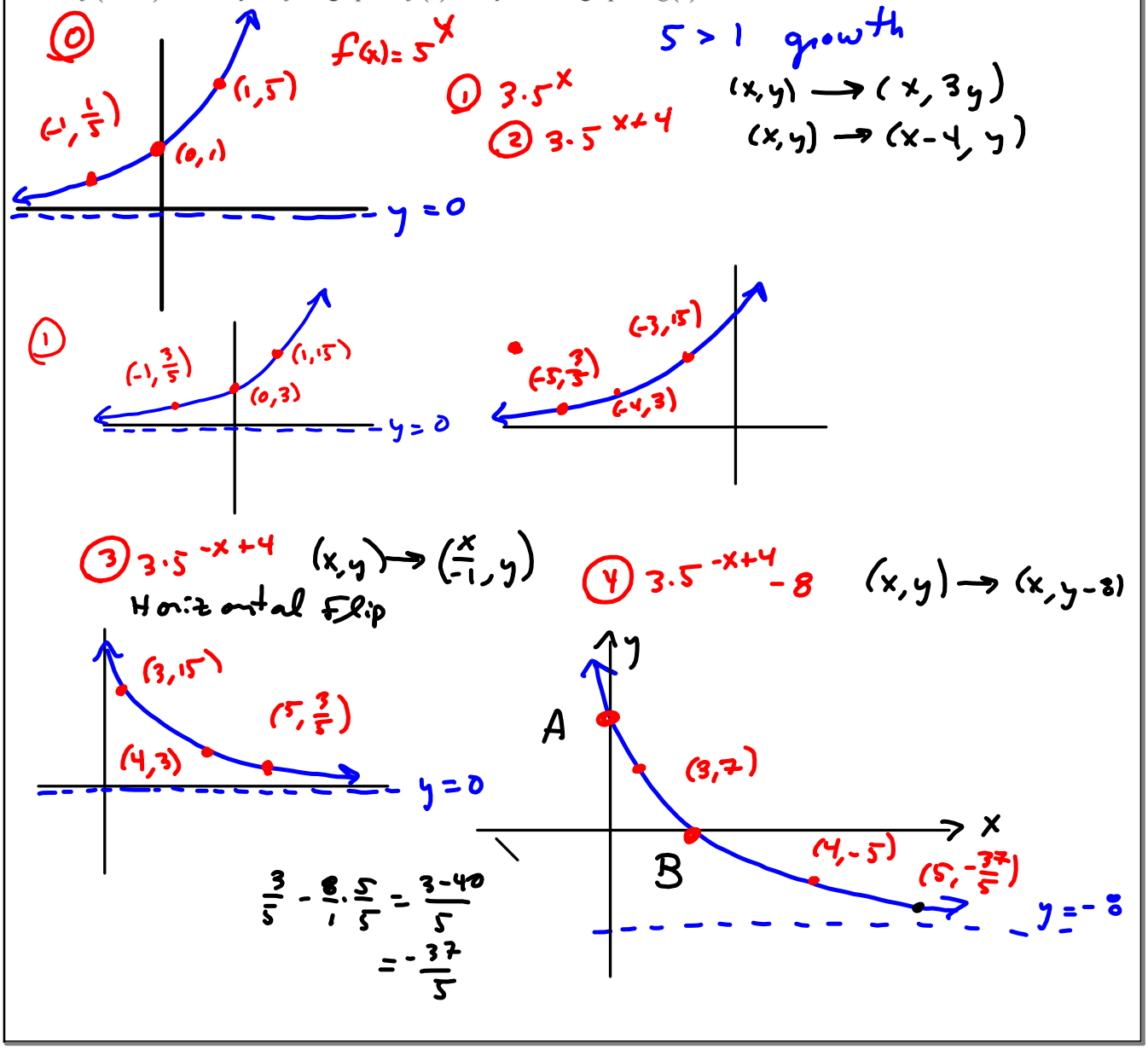
1. (20 pts) Starting with $f(x)=5^x$, sketch the graph of $g(x)=3\cdot 5^{-x+4}-8$ in 5 steps (counting $f(x)=6^x$ as the first step). Use $x=-1$, $x=0$, and $x=1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Finding the x- and y-intercepts is #2, so don't worry about them, until #2. Label each sketch as some variation on $f(x)$, for instance, $7\cdot 5^{x-11}-4$ would be $7f(x-11)-4$. *Only* the *first* graph is $f(x)$. *Only* the *last* graph is $g(x)$.



(2) x- & y-ints:

y-int: $g(0) = 3 \cdot 5^{-(0)+4} - 8 = 3 \cdot 5^{4} - 8$

$A = (0, 3 \cdot 5^{4} - 8)$

x-int: $3 \cdot 5^{-x+4} - 8 = 0$

$3 \cdot 5^{-x+4} = 8$

$5^{-x+4} = \frac{8}{3}$

$\ln(5^{-x+4}) = \ln\left(\frac{8}{3}\right) = D$

$(-x+4)\ln(5) = D$

$(-x+4)E = D$

$-Ex + 4E = D$

$-Ex = D - 4E$

$x = \frac{D-4E}{-E} = \frac{\ln\left(\frac{8}{3}\right) - 4\ln(5)}{-\ln(5)}$

$B = \left(\frac{\ln\left(\frac{8}{3}\right) - 4\ln(5)}{-\ln(5)}, 0\right)$

$A^B = C$

A: $\ln(A^B) = \ln C$

$B\ln A = \ln C$

$B = \frac{\ln C}{\ln A}$

B: $\log_A(A^B) = \log_A(C)$

$B = \log_A(C)$

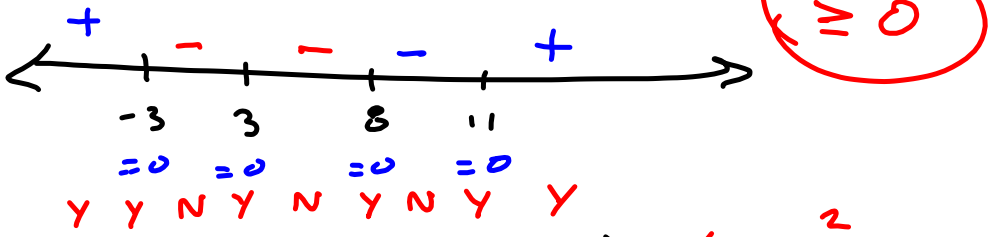
$= \frac{\ln(C)}{\ln(A)}$

Domain of:

$\sqrt{(x-3)^2(x+3)(x-8)^2(x-11)}$.

Need $(x-3)^2(x+3)^1(x-8)^2(x-11)^1 \geq 0$

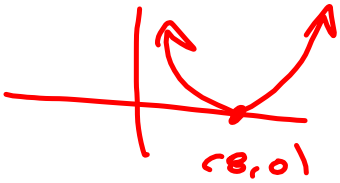
$= x^6 + \cdots$ ↑ ... ↑



Y Y N Y N Y N Y Y

$(-\infty,-3] \cup \{3\} \cup \{8\} \cup [11,\infty)$

$(x-8)^2$



Domain of

$\log_3\left((x-3)^2(x+3)(x-8)^2(x-11)\right)$

Need $(x-3)^2(x+3)(x-8)^2(x-11) > 0$

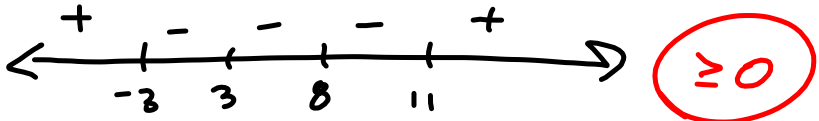
Y N N N N N N N Y

$(-\infty,-3) \cup (11,\infty)$

Domain of

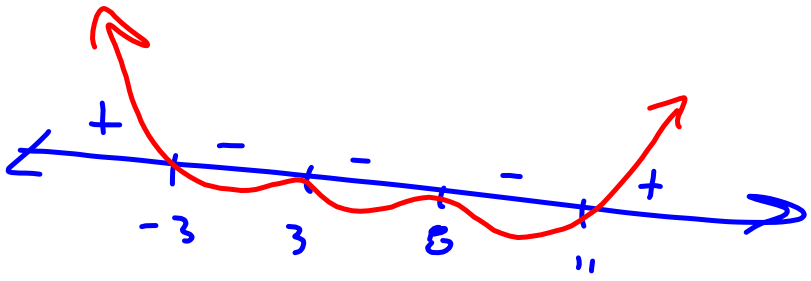
$\sqrt{\dfrac{(x-3)^2(x+3)}{(x-8)^2(x-11)}}$

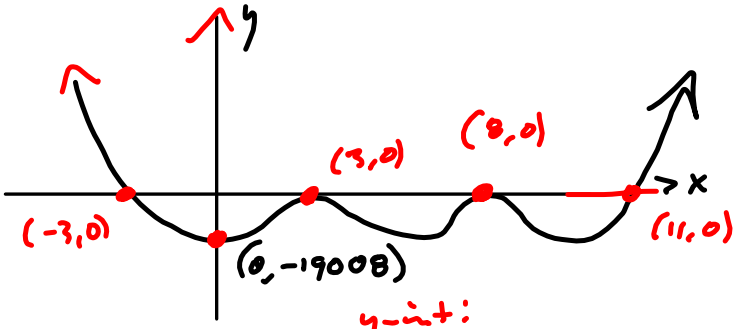
Need $\dfrac{(x-3)^2(x+3)}{(x-8)^2(x-11)} \geq 0$



Y Y N Y N N N N Y

Graph $(x-3)^2(x+3)(x-8)^2(x-11)$

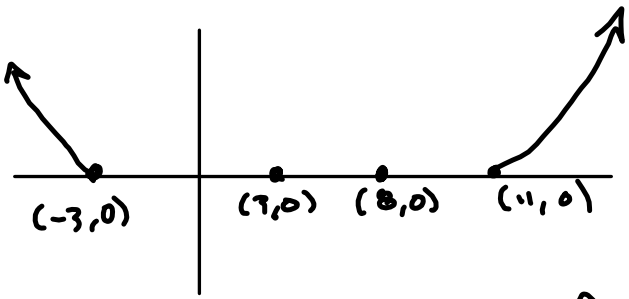




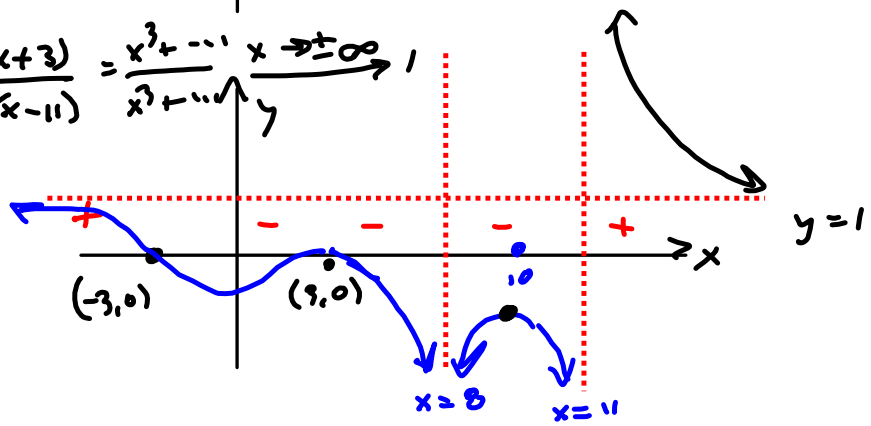
y-int:

$(-3)^2(3)(-8)^2(-11) = -19008$

$\sqrt{(x-3)^2(x+3)(x-8)^2(x-11)}$.



$\frac{(x-3)^2(x+3)}{(x-8)^2(x-11)} = \frac{x^3+\cdots}{x^3+\cdots}$ $x \to \pm\infty$ 1



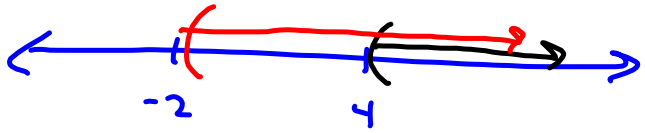
5. Consider the equation $\ln(x-4)+\ln(x+2)=\ln(7)$

   a. (5 pts) What is the domain of this equation?

   b. (5 pts) Solve the equation.

Need $x-4>0$ and $x+2>0$

(a)



AND

N N N N Y

$(4,\infty)$

(b) $\ln\left((x-4)(x+2)\right)=\ln 7$

$e^{\ln(7)}=7$

$e^{\ln(m)}=e^{\ln(n)}$

$(x-4)(x+2)=7$

$x^2-2x-8=7$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x\in\{-3,5\}$

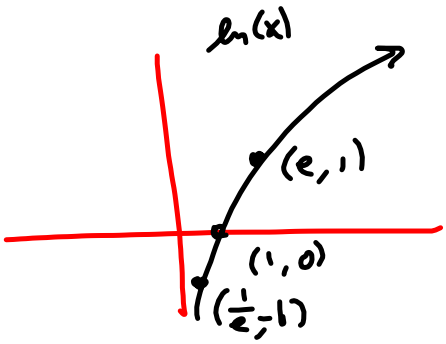
$\boxed{x=5}$ is only soln.

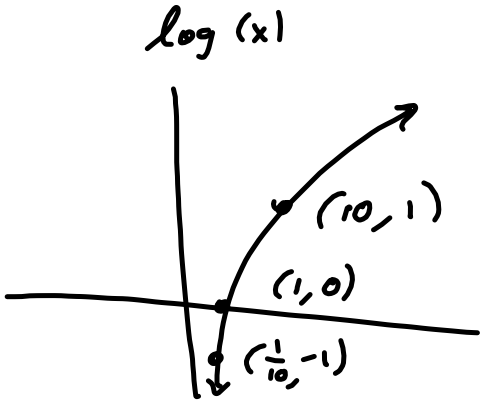
$-3\notin D$

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

$$\log_5(7) = \frac{\ln(7)}{\ln(5)} = \frac{\log(7)}{\log(5)}$$

Base 5 $\leftrightarrow$ Natural $\leftrightarrow$ common





$\log_5(7)$ says write 7 as a power of 5. Report the power

$$(a^b)^c = a^{bc}$$

$$a^b a^c = a^{b+c} \qquad \log(a) + \log(b) = \log(ab)$$

$$a^0 = 1 \qquad \log(1) = 0$$

$$a^{-b} = \frac{1}{a^b} \qquad \log\left(\frac{1}{2}\right) = -\log(2)$$

$$a(b+c) = ab + ac$$

$$ab + ac = a(b+c)$$