① $f(x) = 2x^5 + x^4 - 6x^3 + 16x^2 + 8x - 48$

$2x^5$

f(0) = -48 for graph

② $f(x) = 2x^5 + \underbrace{x^4 - 6x^3}_{1} \underbrace{+ 16x^2}_{2} \underbrace{+ 8x - 48}_{3}$

$\boxed{\text{3 or 1 pos. roots}}$

$f(-x) = -2x^5 + x^4 + 6x^3 + 16x^2 - 8x - 48$

$\boxed{\text{2 or 0 neg. roots.}}$

③ $\frac{p's}{q's}$ "=" $\frac{48}{2}$

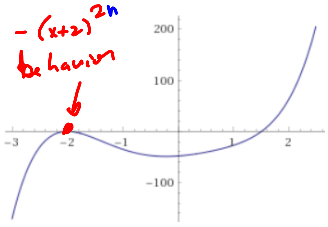
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48,$

$\pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \pm \frac{3}{2}, \cancel{\pm \frac{4}{2}}, \cancel{\pm \frac{6}{2}}, \cancel{\pm \frac{12}{2}}, \cancel{\pm \frac{16}{2}},$

$\cancel{\pm \frac{24}{2}}, \cancel{\pm \frac{48}{2}}$ 12 of 'em!

④

$-(x+2)^{2n}$ behavior



Looks like

$x = -2$, $m = 2n$ = even → kisses x-axis.

$x = \frac{3}{2}$, m = odd

$$
\begin{array}{r|rrrrrr}
-2 & 2 & 1 & -6 & 16 & 8 & -48 \\
 & & -4 & 6 & 0 & -32 & 48 \\
\hline
-2 & 2 & -3 & 0 & 16 & -24 & 0 \\
 & & -4 & 14 & -28 & 24 & \\
\hline
-2 & 2 & -7 & 14 & -12 & 0 & \\
 & & -4 & 22 & & & \\
\hline
 & 2 & -11 & 36 & & &
\end{array}
$$

sweet! Sweet! Nope

$(x+2)^2(2x^3 - 7x^2 + 14x - 12)$ → Depressed Polynomial.

$$\begin{array}{r|rrrr} \frac{3}{2} & 2 & -7 & 14 & -12 \\ & & 3 & -6 & 12 \\ \hline & 2 & -4 & 8 & 0 \\ \hline & x^2 & x & c & r \end{array}$$

sweet!

So we're down to solving

$2x^2-4x+8=0$

$\Rightarrow 2(x^2-2x+4)=0$

$\Rightarrow x^2-2x+4$

$a=1,\ b=-2,\ c=4$

$b^2-4ac = (-2)^2-4(1)(4)$

$=4-16=-12<0$

No real roots!

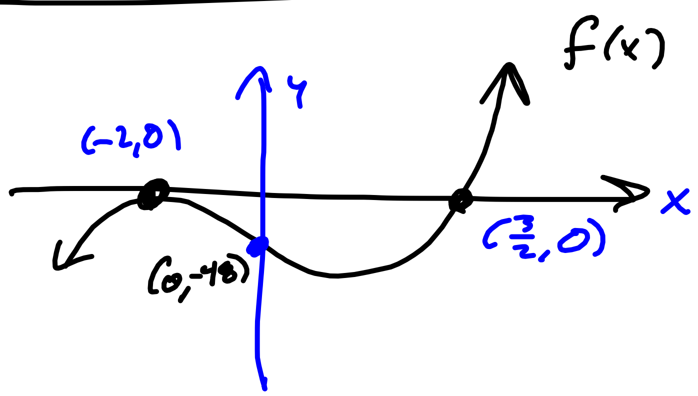
#4: we're keepin' things real.

So we're done

$$\boxed{\begin{array}{l} x=-2,\ m=2 \\ x=\frac{3}{2},\ m=1 \end{array}}$$

5. $\boxed{f(x) = (x+2)^2\left(x-\frac{3}{2}\right)(2x^2-4x+8)}$

6.



7 Handle the nonreal.

$x^2 - 2x + 4 = 0 \Rightarrow b^2 - 4ac = -12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$2 \lfloor 12$
$2 \lfloor 6$
3

$$\sqrt{-2 \cdot 2 \cdot 3} = i2\sqrt{3} = 2i\sqrt{3}$$

$$= \frac{2 \pm 2i\sqrt{3}}{2(1)} = \frac{\cancel{2}(1 \pm i\sqrt{3})}{\cancel{2}} = \boxed{1 \pm i\sqrt{3}}$$

$$\Rightarrow f(x) = 2(x+2)^2\left(x - \tfrac{3}{2}\right)\left(x - (1+i\sqrt{3})\right)\left(x - (1-i\sqrt{3})\right)$$

$= 2x^5$ + smaller stuff

8. $R(x) = \dfrac{x^2+3x-10}{2x^2-13x+15} = \dfrac{(x+5)(x-2)}{(2x-15)(x+1)}$

Nope

$(x+1)(2x-15) = 2x^2-15x+2x-15$

$= 2x^2-13x-15$

$R(x) = \dfrac{(x+5)(x-2)}{(2x-3)(x-5)}$

$\mathscr{D} = \mathbb{R} \setminus \{\frac{3}{2}, 5\}$

$\boxed{V.A.: \; x = 5, \; x = \frac{3}{2}}$

$2x-3=0$

$2x=3$

$x=\frac{3}{2}$

etc.

y-int: $R(0) = \frac{-10}{15} = -\frac{2}{3}$

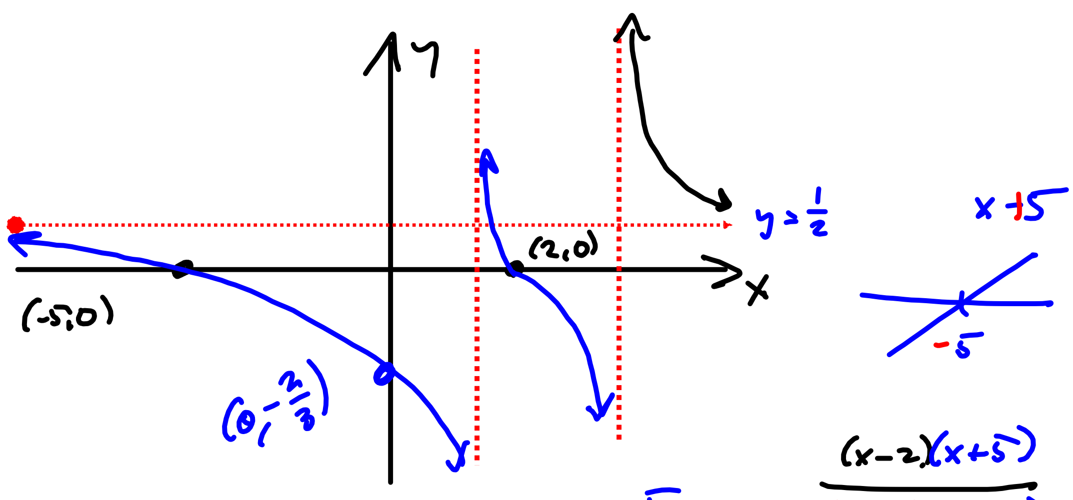
$\boxed{(0, -\frac{2}{3})}$

$\boxed{H.A.: \; y = \frac{1}{2}}$

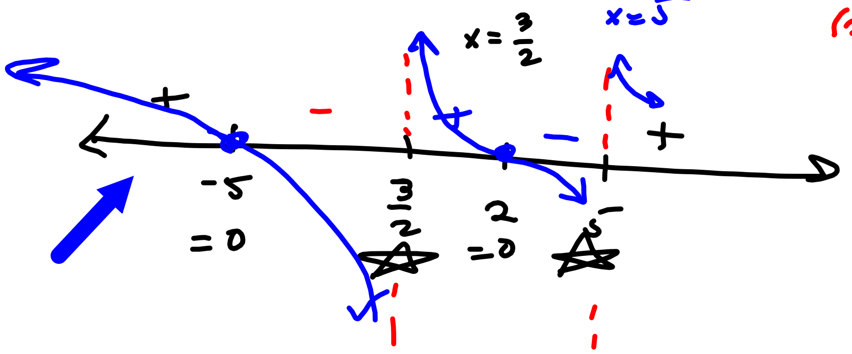
x-int: $R(x) = 0 \rightarrow$

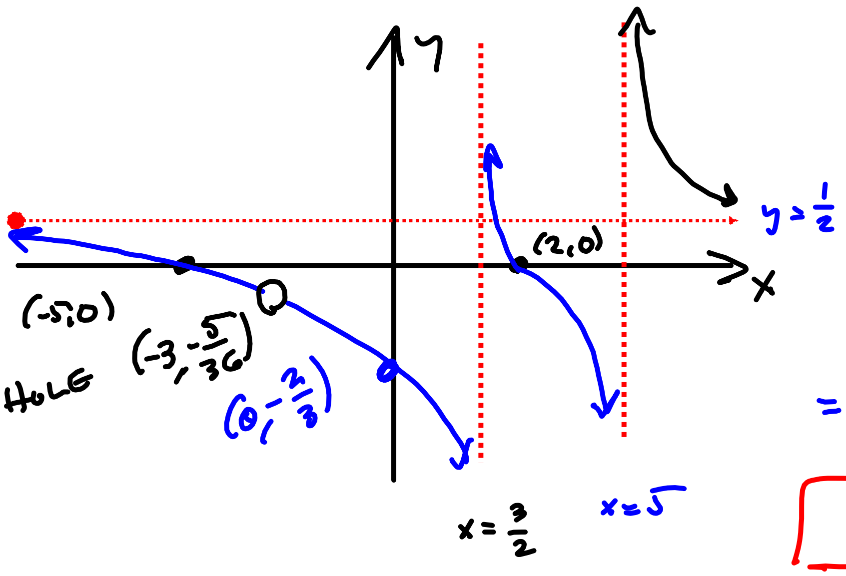
$x = -5, 2 \rightarrow$

$\boxed{\text{x-int}}$ $(-5,0), (2,0)$



$\dfrac{(x-2)(x+5)}{(2x-3)(x-5)}$





Hole @ $x = -3$

$$R(-3) = \frac{(-3+5)(-3-2)}{(2(-3)-3)(-3-5)} = \frac{\cancel{2}(-5)}{(-9)(-\cancel{8})} = -\frac{5}{36} \Rightarrow$$

$$\boxed{\left(-3, -\frac{5}{36}\right) \text{ is hole!}}$$

$$Q(x) = \frac{x^3 + 6x^2 - x - 30}{2x^3 - 7x^2 - 24x + 45} = \frac{(x^2+3x-10)(x-c)}{(2x^2-13x+15)(x-c)}$$

How to find c? Find R(x) inside Q(x).

$$= \frac{(x-2)(x+5)(x-c)}{(2x-3)(x-5)(x-c)} = \frac{(x-2)(x+5)(x+3)}{(2x-3)(x-5)(x+3)}$$

2	1	6	-1	-30
		2	16	30
-5	1	8	15	0
		-5	-15	
	1	3		

$x+3$

That

Check: $x+3$ a factor of denominator?

-3	2	-7	-24	45
		-6	39	-45
	2	-13	15	0

Sweet!

(10) $\frac{x^3-7x^2-9x+63}{x^2-6x+5} = \frac{x^2(x-7)-9(x-7)}{(x-5)(x-1)}$

$= \frac{(x-7)(x^2-9)}{(x-5)(x-1)} = \frac{(x-7)(x-3)(x+3)}{(x-5)(x-1)}$

$\mathcal{D} = \mathbb{R} \setminus \{1,5\}$

$\boxed{\text{V.A.: } x=1,\ x=5}$

$\frac{x^3}{x^2} \Rightarrow$ Oblique Asymptote

$\boxed{\begin{array}{l} \text{y-int: } (0, \frac{63}{5}) \\ \text{x-int: } (-3,0), (3,0), (7,0) \end{array}}$

$\boxed{y = x-1 \text{ is O.A.}}$

$$\begin{array}{r} x-1 \\ x^2-6x+5 \overline{) x^3-7x^2-9x+63} \\ -(x^3-6x^2+5x) \\ \hline -x^2-14x \end{array}$$

$\frac{x^3}{x^2} = x$

$\frac{-x^2}{x^2} = -1$

$x - 1$

$\cancel{-(\ \ )}$

$-20x + 69$

Divide it all the way & get $x-1$, r $-20x+69$

This says $T(x) = \frac{x^3-7x^2-9x+63}{x^2-6x+5}$

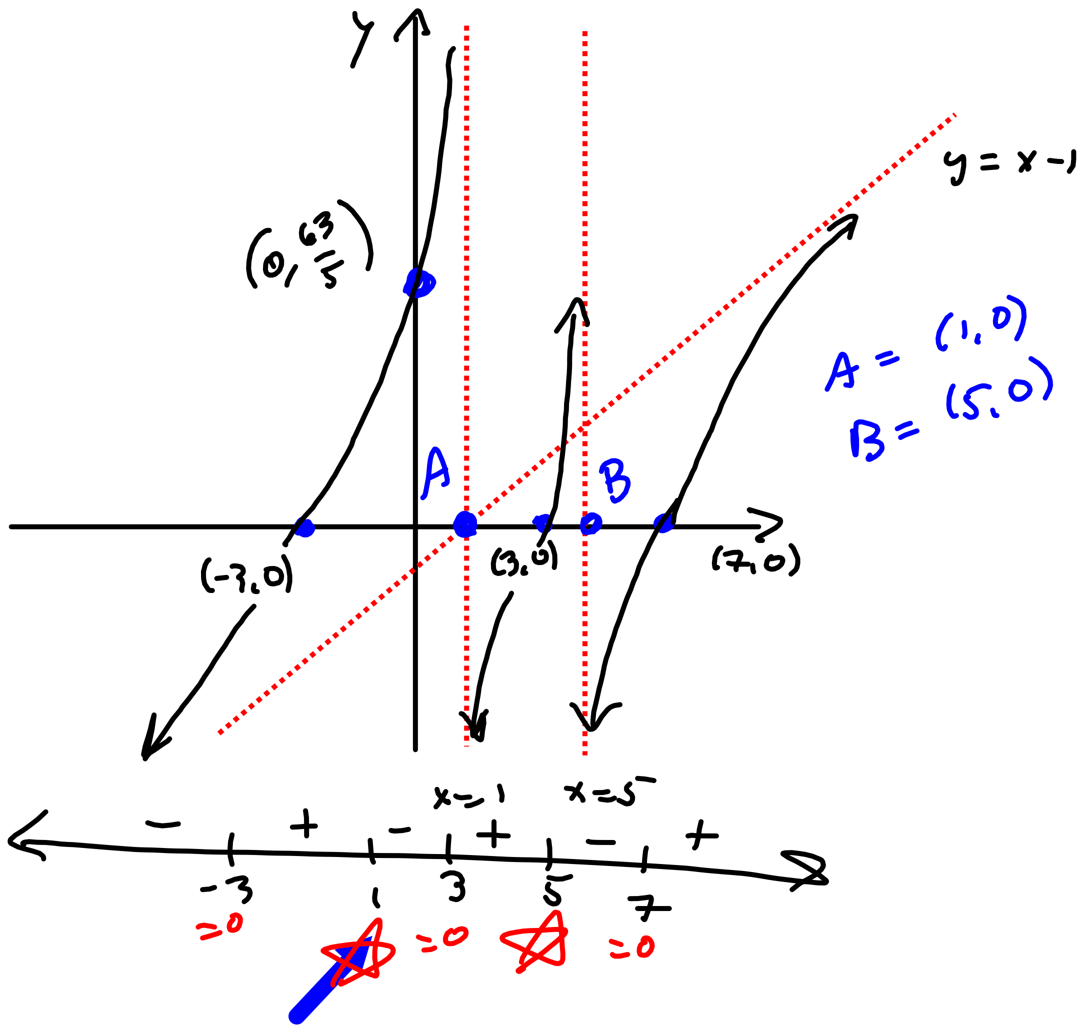
$= x-1 + \frac{-20x+69}{x^2-6x+5}$

$\boxed{V.A.: \ x=1, x=5}$

$\boxed{y=x-1 \text{ is O.A.}}$

y-int: $(0, \frac{63}{5})$

x-int $(-3,0), (3,0), (7,0)$



11) $w(x) = \sqrt{(x-5)(x+1)(x-2)^2(x+7)}$

$\mathcal{D}(w) = \{x \mid (x-5)(x+1)(x-2)^2(x+7) \geq 0\}$

$x^5 +$ stuff

$-$ $+$ $-$ $-$ $+$ ≥ 0

$=0$ $=0$ $=0$ $=0$

-7 -1 2 5

N Y Y Y N Y N Y Y

$= \boxed{[-7,-1] \cup \{2\} \cup [5,\infty)}$

12) Need $\dfrac{(x-2)^2(x+1)}{(x-5)(x+7)} \geq 0$

$-$ $+$ $-$ $-$ $+$ ≥ 0

-7 -1 2 5

$\nexists$ $=0$ $=0$ $\nexists$

$= \boxed{(-7,-1] \cup \{2\} \cup (5,\infty)}$