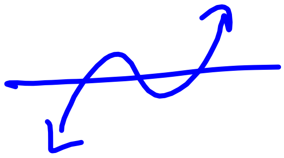
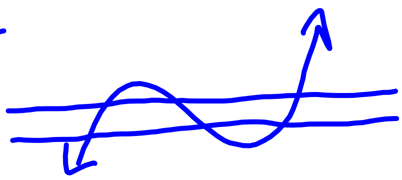
$2x^3 - 3x^2 - 50x + 18$

Check $x = -5$. Look for alternating signs

$$\begin{array}{r|rrrr} -5 & 2 & -3 & -50 & 18 \\ & & -10 & 65 & -75 \\ \hline & 2 & -13 & 15 & -\text{something} \end{array}$$

Alternating signs
$\therefore$ Lower bound!





-2

$expand((x-1)^2\cdot(x+7)\cdot(x-2+\mathrm{sqrt}(3)\cdot I)\cdot(x-2-\mathrm{sqrt}(3)\cdot I))$

$$x^5+x^4-26x^3+94x^2-119x+49$$

Descartes:

Rational zeros: $\pm1, \pm7, \pm49$

$$\begin{array}{r|rrrrrr} 1 & 1 & 1 & -26 & 94 & -119 & 49 \\ & & 1 & 2 & -24 & 70 & -49 \\ \hline & 1 & 2 & -24 & 70 & -49 & 0 \end{array}$$ sweet!

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & -24 & 70 & -49 \\ & & 1 & 3 & -21 & 49 \\ \hline & 1 & 3 & -21 & 49 & 0 \end{array}$$ sweet!

$$\begin{array}{r|rrrr} 1 & \boxed{1_{x^3} \quad 3_{x^2} \quad -21_x \quad 49_c} \\ & \quad 1 \quad 4 \quad -17 \\ \hline & 1 \quad 4 \quad -17 \quad \text{Neeup} \end{array}$$

Here's where we're at:

$(x-1)^2(x^3+3x^2-21x+49)$

$$\begin{array}{r|rrrr} -7 & 1 & 3 & -21 & 49 \\ & & -7 & 28 & -49 \\ \hline & 1 & -4 & 7 & 0 \end{array}$$ sweet!

$(x-1)^2(x+7)(\underline{x^2-4x+7})$

$x^2-4x+2^2-4+7 = (x-2)^2+3 = 0$

$(x-2)^2=-3$

$x-2=\pm\sqrt{3} = \pm i\sqrt{3}$

$x=2\pm i\sqrt{3}$

$a=1,\ b=-4,\ b=7$

$b^2-4ac = (-4)^2-4(1)(7) = 16-28 = -12$

$$\begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ & 3 \end{array} \qquad \sqrt{12}=2\sqrt{3}$$

$x=\dfrac{+4\pm\sqrt{-12}}{2(1)} = \dfrac{4\pm2i\sqrt{3}}{2}$

$= \dfrac{2(2\pm i\sqrt{3})}{2} = 2\pm i\sqrt{3}$

This says $f(x)=(x-1)^2(x+7)(x-2+i\sqrt{3})(x-2-i\sqrt{3})$

Split into linear factors, over the Complex #s.

Factored over the real #s, we knew we were done when $b^2-4ac=-12$

$(x-1)^2(x+7)(x^2-4x+7)$ is as far as the real #s take us.

$\underline{x^2-4x+7}$ is irreducible over the reals.

No real roots, so can't factor this, just using real #s.