Recall: Synthetic Division:

$f(x) = x^5 - 3x^4 + x^3 + 11x^2 - 18x + 8$

Divide by $g(x) = x - 2$:

$$
\begin{array}{r|rrrrrl}
2 & 1 & -3 & 1 & 11 & -18 & 8 \\
  &   & 2 & -2 & -2 & 18 & 0 \\
\hline
  & 1 & -1 & -1 & 9 & 0 & \boxed{8 = f(2)}
\end{array}
$$

Remainder Theorem.

This says that

$f(x) = (x-2)(x^4 - x^3 - x^2 + 9x) + 8$

$\frac{37}{5} = 7 + \frac{2}{5}$

$37 = 5 \cdot 7 + 2$

What's $f(2)$?

$(2-2)(2^4 - 2^3 - 2^2 + 9(2)) + 8 = 8.$

$f(x) = x^5 - 3x^4 + x^3 + 11x^2 - 18x + 8$

Rational zeros Theorem gives us guesses to make.

Cut to the chase: $x = 1, -2$ are good guesses.? I want to find all roots (zeros) of $f(x)$ & graph it.

$$\begin{array}{r|rrrrrr} 1 & 1 & -3 & 1 & 11 & -18 & +8 \\ & & 1 & -2 & -1 & 10 & -8 \\ \hline 1 & 1 & -2 & -1 & 10 & -8 & 0 \\ & & 1 & -1 & -2 & 8 & \\ \hline 1 & 1 & -1 & -2 & 8 & 0 & \\ & & 1 & 0 & -2 & & \\ \hline & 1 & 0 & -2 & 6 & & \end{array}$$

Sweet! This says $f(x) = (x-1)(x^4 - 2x^3 - x^2 + 10x - 8)$

Sweet!

$= (x-1)^2 \underbrace{(x^3 - x^2 - 2x + 8)}_{\text{New depressed polynomial.}}$

6 Nope

Try $x = -2$ on this $\longrightarrow$

$$\begin{array}{r|rrrr} -2 & 1 & -1 & -2 & 8 \\ & & -2 & 6 & -8 \\ \hline & 1 & -3 & 4 & 0 \end{array}$$

sweet!

This says $(x-1)^2(x+2)\underbrace{(x^2 - 3x + 4)}_{\text{Quadratic!}}$

$x^2 - 3x + 4 = 0$

$a = 1, b = -3, c = 4$

$b^2 - 4ac = (-3)^2 - 4(1)(4) = 9 - 16 = -7$ ← Doesn't factor by ac

NOT REAL

$x = \dfrac{3 \pm \sqrt{-7}}{2} = \dfrac{3 \pm i\sqrt{7}}{2}$

So, $f(x) = (x-1)^2(x+2)\left(x - \dfrac{3+i\sqrt{7}}{2}\right)\left(x - \dfrac{3-i\sqrt{7}}{2}\right)$

WE'VE SPLIT $f(x)$ into linear factors. LINEAR MEANS x^1

LINEAR FACTOR

$(x - c)$ OR $(x - r)$

OR $(bx + c)$ $(3x + 2)$

So, $f(x) = (x-1)^2(x+2)\boxed{\left(x-\frac{3+i\sqrt{7}}{2}\right)\left(x-\frac{3-i\sqrt{7}}{2}\right)}$

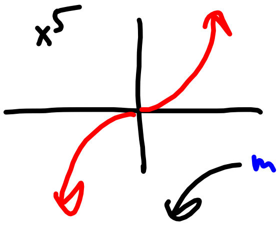
No effect for our graphs.

What's it look like?

$f(x) = x^5 +$ lower degree

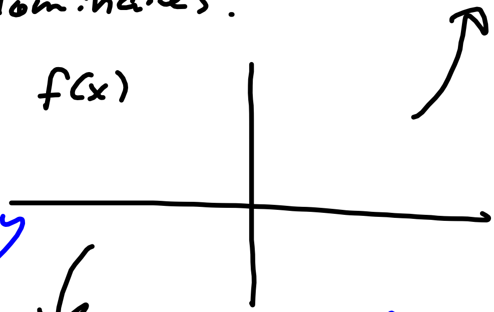
Eventually x^5 dominates.

x^5



so

$f(x)$

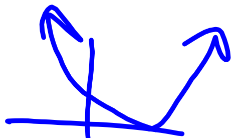


multiplicity

$x=1, m=2$ TOUCH

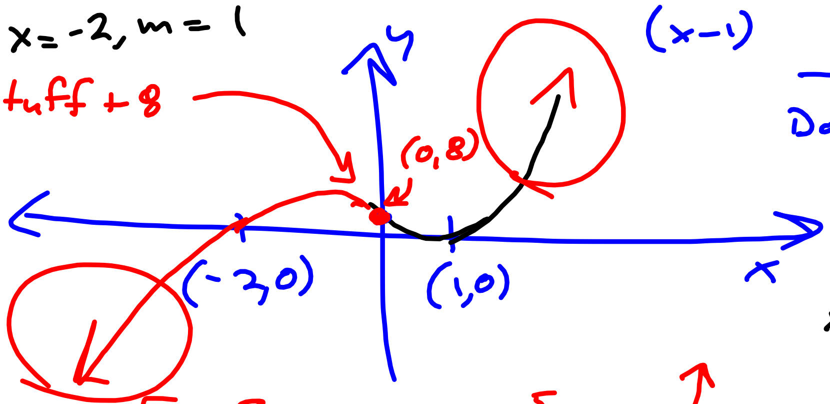
$x=-2, m=1$

$(x-1)^2$



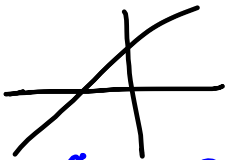
Does not cross x-axis

$f(x) =$ stuff $+ 8$



Theorems: END BEHAVIOR x^5 .....

$x+2$



Crosses x-axis

Remainder, Factor Theorem, Descartes' Rule of Signs, Rational Zeros

Fundamental Theorem of Algebra.

It says if $f(x)$ is degree n, then $f(x)$ has n zeros. (Not Necessarily real, and we count repetitions in that number.)

$\boxed{x=1, m=2}$

$x=-2, m=1$

$x=\frac{3+i\sqrt{7}}{2}, m=1$

$x=\frac{3-i\sqrt{7}}{2}, m=1$

Degree $= 5$

Counts as 2, so 5 roots.

Application:

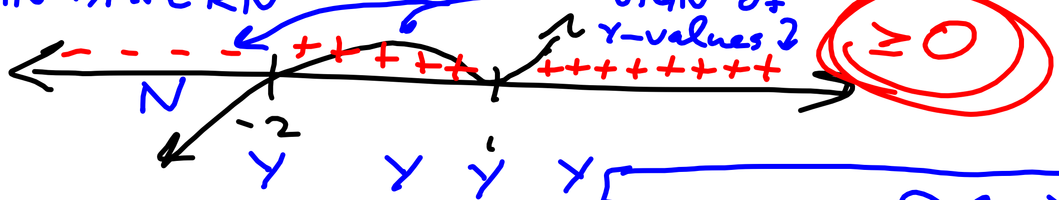
what's the domain of $\sqrt{f(x)}$?

Need $f(x) \geq 0$

$$(x-1)^2(x+2)\left(x-\frac{3+i\sqrt{7}}{2}\right)\left(x-\frac{3-i\sqrt{7}}{2}\right) \geq 0$$

SIGN PATTERN

SIGN OF Y-values?



So, $f(x) \geq 0$ when $x \in \boxed{[-2, \infty) = \mathcal{D}(\sqrt{f})}$

ANALYZED FOR "≥ 0" condition.