Geometric Series

n of 'em

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-2} + ar^{n-1} = \sum_{k=1}^{n} ar^{k-1}$$

$1 + 2 + 4 + 8 + \cdots + 256$

$a = 1,\ r = 2,\ n = 9$ terms.

$= 1 + 2 + 2^2 + 2^3 + \cdots + 2^8 = \sum_{k=1}^{9} 1 \cdot 2^{k-1} = 1\left(\frac{1-2^9}{1-2}\right)$

$1 \cdot 2^0$ (k=1), $1 \cdot 2^1$ (k=2)

$= \frac{511}{1} = 511$

$64 = 2^6$

$128 = 2^7$

$256 = 2^8$

$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-2} + ar^{n-1}$

$-rS_n = -(ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} + ar^n)$

$S_n - rS_n = S_n(1-r) = a - ar^n = a(1-r^n)$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Simple, ordinary annuity certain

Fixed time t yrs $\quad R$ = periodic payment

Pay @ end of the month

r = annual interest rate is fixed

m = # of payments & # of compounding periods.

n = total # of periods $= mt$

i = interest rate per period $= \frac{r}{m}$

Recall Compound Interest.

$$A(t) = \boxed{P(1+i)^n} = P\left(1+\frac{r}{m}\right)^{mt}$$

Principal

$$R(1+i)^{n-1} + R(1+i)^{n-2} + \dots + R(1+i) + R$$

$$R + R(1+i) + R(1+i)^2 + \dots + R(1+i)^{n-2} + R(1+i)^{n-1}$$

Common ratio is $(1+i)$ = the "r" in geometric series (Not interest rate)

The 'a' is $a = R$

$$\sum_{k=1}^{n} ar^{n-1} = a\left(\frac{1-r^n}{1-r}\right) = R\left(\frac{1-(1+i)^n}{1-(1+i)}\right) = R\left(\frac{1-(1+i)^n}{-i}\right)$$

$1-(1+i) = -i$

Future Value of an Annuity.

$$S_n = R\left(\frac{(1+i)^n - 1}{i}\right)$$

Car Loan is an annuity the banker "buys" from you!

Banker wants a particular rate of return.

3-yr loan .. 12% interest compounded monthly.

$$A = P\left(1+\frac{.12}{12}\right)^{12\cdot 3} = R\left(\frac{\left(1+\frac{.12}{12}\right)^{36}-1}{\frac{.12}{12}}\right)$$

Amt Borrowed → Pmt.

Amortization → Payment calculator

Formula from $P(1+i)^n = R\left(\frac{(1+i)^n-1}{i}\right)$

$$\frac{(1+i)^n[Pi]}{(1+i)^n[1-(1+i)^{-n}]} = \frac{Pi(1+i)^n}{(1+i)^n-1}$$

$$R = \frac{Pi}{1-(1+i)^{-n}}$$ PMT

$\frac{(1+i)^n}{(1+i)^n} = 1 \qquad \frac{1}{(1+i)^n} = (1+i)^{-n}$

$\frac{1}{2^3} = 2^{-3}$

How much money in the bank after 5 years of $600/mo pmts into an account that earns 5% compounded monthly. (Pmts @ end of month.)

$$FV = R\left(\frac{(1+i)^n - 1}{i}\right) = R\left(\frac{\left(1+\frac{r}{m}\right)^{mt} - 1}{(r/m)}\right)$$

$\approx \$40,803.65$

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600*((1+.05/12)^
(12*5)-1)/(.05/1
2)
        40803.6497
600*12*5
            36000
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→ No interest result. Helps you know if your answer makes sense.