

Geometric Series ^{n of term}

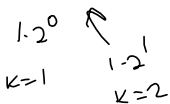
$$a + ar + ar^2 + ar^3 + \dots + ar^{n-2} + ar^{n-1} = \sum_{k=1}^n ar^{k-1}$$

$$1 + 2 + 4 + 8 + \dots + 256$$

$a=1, r=2, n=9$ terms.

$$= 1 + 2 + 2^2 + 2^3 + \dots + 2^8 = \sum_{k=1}^9 1 \cdot 2^{k-1} = 1 \left(\frac{1-2^9}{1-2} \right) = \frac{511}{1} = 511$$

64 = ~~2~~ 2^6
 128 = ~~2~~ 2^7
 256 = 2^8



~~$$\sum_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-2} + ar^{n-1}$$~~
~~$$- \sum_n = - (ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-2} + ar^{n-1} + ar^n)$$~~

$$\sum_n - r \sum_n = \sum_n (1-r) = a - ar^n = a(1-r^n)$$

$$\sum_n = \frac{a(1-r^n)}{1-r}$$

Simple, ordinary annuity certain
 Fixed time t yrs $R =$ periodic payment
 Pay @ end of the month
 $r =$ annual interest rate is fixed
 $n =$ # of payments & # of compounding periods.
 $n =$ total # of periods $= mt$
 $i =$ interest rate per period $= \frac{r}{m}$

Recall Compound Interest.
 $A(t) = P(1+i)^n = P(1+\frac{r}{m})^{mt}$
 Principal

$R(1+i)^{n-1} + R(1+i)^{n-2} + \dots + R(1+i) + R$
 $R + R(1+i) + R(1+i)^2 + \dots + R(1+i)^{n-2} + R(1+i)^{n-1}$
 Common ratio is $(1+i)$ = the "r" in geometric series
 (Not interest rate)

The 'a' is $a = R$

$$\sum_{k=1}^n ar^{n-k} = a \left(\frac{1-r^n}{1-r} \right) = R \left(\frac{1-(1+i)^n}{1-(1+i)} \right) = R \left(\frac{1-(1+i)^n}{-i} \right)$$

Future Value of an Annuity.
 $S_n = R \left(\frac{(1+i)^n - 1}{i} \right)$

$$1 - (1+i) = -i$$

Can Loan is an annuity the banker "buys" from you!
 Banker wants a particular rate of return.

3-yr loan ... 12% interest compounded monthly.

$$A = P \left(1 + \frac{.12}{12} \right)^{12 \cdot 3} = R \left(\frac{\left(1 + \frac{.12}{12} \right)^{36} - 1}{\frac{.12}{12}} \right)$$

Am't Borrowed \rightarrow Pmt.

Amortization \rightarrow Payment calculator

Formula from $P(1+i)^n = R \left(\frac{(1+i)^n - 1}{i} \right)$

$$\frac{(1+i)^n [P]}{(1+i)^n [1 - (1+i)^{-n}]} = \frac{P(1+i)^n}{(1+i)^n - 1} \Rightarrow R = \frac{Pi}{1 - (1+i)^{-n}}$$

PMT

$$\frac{(1+i)^n}{(1+i)^n} = 1 \quad \frac{1}{(1+i)^n} = (1+i)^{-n}$$

$$\frac{1}{2} = 2^{-3}$$

How much money is in the bank after 5 years of \$600/mo pmts into an account that earns 5% compounded monthly. (Pmts @ end of month.)

$$FV = R \left(\frac{(1+i)^n - 1}{i} \right) = R \left(\frac{(1+\frac{r}{m})^{mt} - 1}{(r/m)} \right)$$

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600*((1+.05/12)^
(12*5)-1)/(.05/1
2)
40803.6497
600*12*5      36000
  
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$$\approx \$40,803.65$$

→ No interest result.
Helps you know if your answer makes sense.