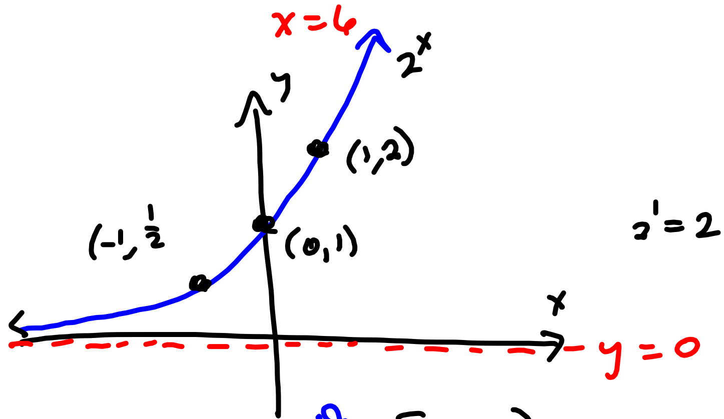
$2^x = 64 = 2^6$

$$\begin{array}{r|l} 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$



$2^1 = 2$

$\sqrt{x+2}$ Need $x+2 \geq 0$

$\mathcal{D} = \{x \mid x \geq -2\}$

$\mathcal{D} = [-2, \infty)$

$f(x) = y = \boxed{\sqrt{x+2}}$ $\quad R = [0, \infty)$

$\sqrt{x+2} = y$

$\sqrt{y+2} = x$

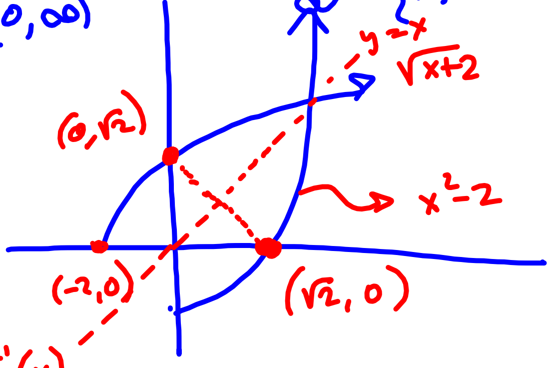
$\left(\sqrt{y+2}^{\,2} = x^2 \right)$

$y + 2 = x^2$

$y = x^2 - 2 = f^{-1}(x)$

$\mathcal{D} = [0, \infty)$

$R = [-2, \infty)$



Inverse of 2^x is $\log_2(x)$

Inverse of $\sqrt{x+2}$ is x^2-2

$f(x)$ $\qquad$ $g(x)$

$$(f \circ g)(x) = f(g(x)) = f(x^2-2) =$$

$$f(\Box) = \sqrt{\Box + 2}$$

$$f(\heartsuit) = \sqrt{\heartsuit + 2}$$

$$f(x^2-2) = \sqrt{(x^2-2)+2}$$

$$= \sqrt{x^2}$$

$$= |x|$$

$= x$, after we restricted the domain of $y = x^2-2$ to $[0, \infty)$

$g(x) = x^2 - 2$

$g(☺) = ☺^2 - 2$

$$(g \circ f)(x) = g(f(x))$$

$$= g(\sqrt{x+2}) = (\sqrt{x+2})^2 - 2$$

$$= x+2-2 = x$$

$f(x) = 2^x$ & $\log_2(x) = g(x)$

They're inverses

$f(g(x)) = f(\log_2(x)) = 2^{\log_2(x)} = x$

$g(f(x)) = g(2^x) = \log_2(2^x) = x$

$\log_2(32) = \log_2(2^5) = 5$

Logs extract variable from the exponent

$2.5^{7x} = 11$

$\log_{2.5}(2.5^{7x}) = \log_{2.5}(11)$

$7x = \log_{2.5}(11)$

$x = \frac{1}{7}\log_{2.5}(11)$

$= \frac{1}{7}\frac{\log(11)}{\log(2.5)}$

Calculator has $\log(x)$ & $\ln(x) = \log_e(x)$

$\boxed{\text{exponential functions grow at a rate proportional to their size}}$

In calculus, $y = 2^x$. Then
$y' =$ growth rate
= slope / steepness.

$y' = k \cdot 2^x$

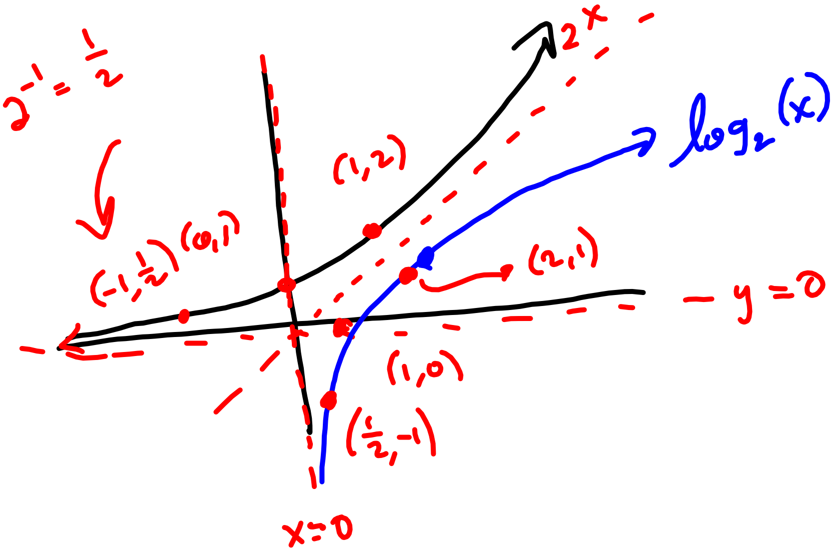
Euler

"e" is special.
It's the <u>one</u> number with $k=1$

$\frac{1}{1000000000000}$

"e" is the NATURAL BASE
for "Euler."

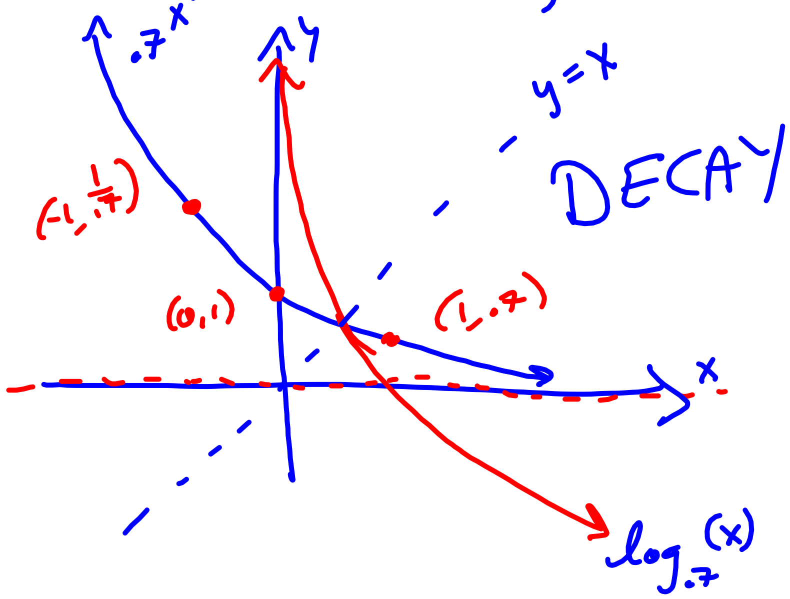
$2^{-1} = \frac{1}{2}$



$\mathcal{D}(2^x) = \mathbb{R}$
$\mathcal{R}(2^x) = (0,\infty)$
$\mathcal{D}(\log_2(x)) = (0,\infty)$
$\mathcal{R}(\log_2(x)) = (-\infty,\infty)$

Exponential growth : b^x , $b>1$

Exponential decay b^x , $0<b<1$



What's the half-life of

$P(t) = 27e^{-.01t} = P_0 e^{-.01t}$ P_0 = original amount.

when is $P(t) = \frac{1}{2} P_0$?

$$27e^{-.01t} = \frac{1}{2}(27) \qquad P_0 e^{-.01t} = \frac{1}{2} P_0$$

$$e^{-.01t} = \frac{1}{2}$$

$$\ln\left(e^{-.01t}\right) = \ln\left(\tfrac{1}{2}\right)$$

$$-.01t = \ln\left(\tfrac{1}{2}\right)$$

$$t = \frac{\ln\left(\frac{1}{2}\right)}{-.01}$$

P = initial investment.

$\frac{10}{100} = \frac{1}{10} = .1$

$P = 100$ $\quad$ 10% simple interest on \$100 is \$10.

A = future amount $= P + I = P + rP$

Simple interest. $= P(1+r)$

Simple interest of 10% per year for 20 years $100 + 100(.1)(20)$

$= 100 + 200 = 300$

$$A = P + Prt$$
$$= P(1+rt)$$

Compound Interest: Simple interest for one period, invest that interest, so interest compounds.

12% annual interest compounded monthly.

$\frac{.12}{12} = .01$

Month	Balance
0	100
1	$100 + .01 \cdot 100 = 100(1+.01)$
2	$\boxed{100(1+.01)} + .01(100(1+.01))$
	$= 100(1+.01)(1+.01) = 100(1+.01)^2$
⋮	
10	$100(1+.01)^{10}$

After n periods, initial amt P, m periods per year, APR = r, $n = mt$

A = future amt

$$A(t) = P\left(1+\frac{r}{m}\right)^n = P\left(1+\frac{r}{m}\right)^{mt}$$

FACT

$m \to \infty \Rightarrow Pe^{rt}$ = continuous compounding!

\$1000, 25yrs, r = .02, m = 365