8. (3 pts) Sketch the graph of $R(x)=\dfrac{x^2-5x-6}{x^2-5x+6}$.

$\dfrac{(x+1)(x-6)}{(x-3)(x-2)}$ HOLE @ $x=3$

Zeros upstairs : x-ints.

.. downstairs: vertical asymptotes

Same zero upstairs & downstairs : HOLE.

$R(x)=\dfrac{(x-6)^1(x+1)}{(x-3)(x-2)}$ → $x=6, x=1$ → $x=2, x=3$

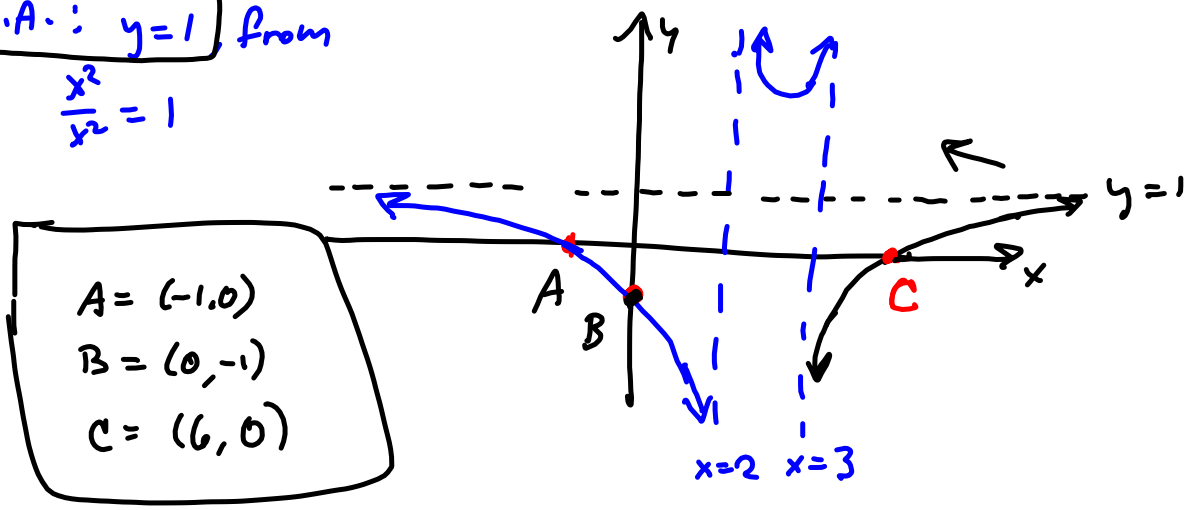
$\mathcal{D} = \mathbb{R}\setminus\{2,3\}$

x-ints : $(6,0)$, $(-1,0)$

V.A. : $x=2$, $x=3$

y-int : $(0,-1)$

H.A. : $y=1$ from $\dfrac{x^2}{x^2}=1$



A = (-1,0)
B = (0,-1)
C = (6,0)

There's an $R(x)=\dfrac{x^2-5x-6}{x^2-5x+6}$. $=\dfrac{(x+1)(x-6)}{(x-3)(x-2)}$ Living inside! We just find the hole!

9. (3 pts) The graph of $g(x)=\dfrac{x^3-9x^2+14x+24}{x^3-9x^2+26x-24}$ differs from the graph of f, in #8, in only one small detail.

Sketch the graph of g, showing all asymptotes, intercepts and holes.

So $g(x)=\dfrac{(x+1)(x-6)(x-4)}{(x-3)(x-2)(x-4)}$

$=g^*(x)=\dfrac{(x+1)(x-6)}{(x-3)(x-2)}$ if $x\neq 4$

$x=4$ is hole!

$x^3-9x^2+14x+24$

$-1 \mid 1 \;-9 \;\; 14 \;\; 24$

$\quad\quad -1 \;\; 10 \;\; -24$

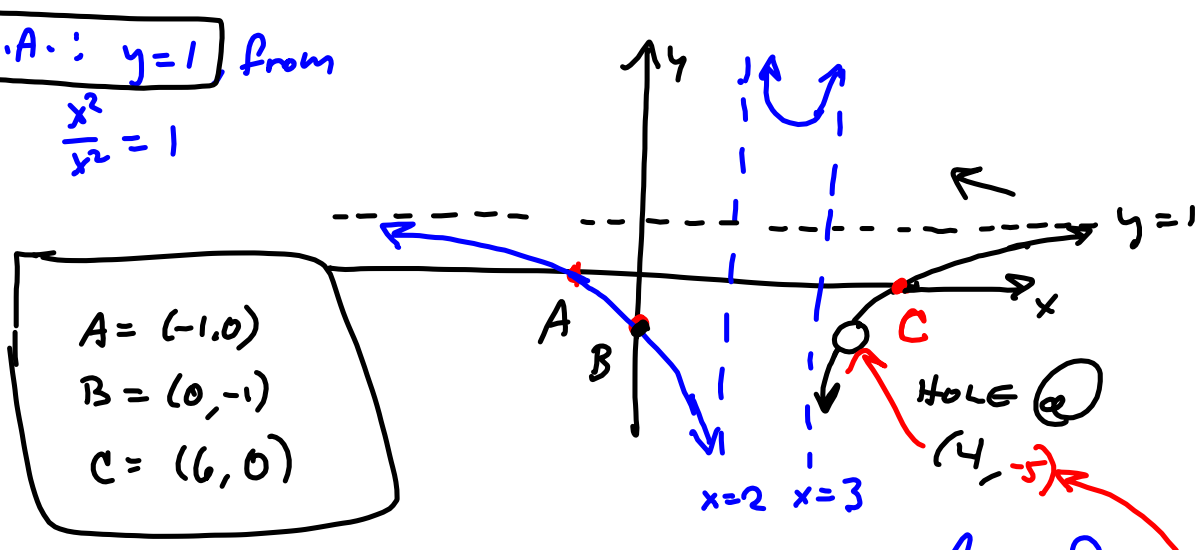
$6 \mid 1 \;-10 \;\; 24 \;\; 0$

$\quad\quad 6 \;\; -24$

$1 \;\; -4 \rightarrow x-4$

$(x+1)(x-6)(x-4)$

H.A.: $y=1$, from $\dfrac{x^2}{x^2}=1$



A = (-1, 0)
B = (0, -1)
C = (6, 0)

$\dfrac{(x+1)(x-6)}{(x-2)(x-3)}$

To locate y-value of the hole

$g^*(4)=\dfrac{(4+1)(4-6)}{(4-2)(4-3)}=\dfrac{-10}{2}=-5$

END Behavior: Rational Func. $\frac{x+1}{x^2-5} \xrightarrow{x\to BIG} 0=y$

$\frac{\text{LOWER Degree}}{\text{HIGHER Degree}}$ = Proper : $y=0$ is H.A.

$\frac{\text{SAME}}{\text{SAME}}$ = IMPROPER. Just look @ highest degree terms

Previous prob. $\frac{x^2-5x-6}{x^2-5x+6} \xrightarrow{x\to BIG} \frac{x^2}{x^2} = 1 = y$

$\frac{\text{Higher Degree}}{\text{Lower Degree}}$ = Improper. Leads to slant asymptote we find by long division.

10. (3 pts) Sketch the graph of $R(x)=\frac{x^3-4x^2-7x+10}{x^2-x-6}$

$\mathcal{D} = \mathbb{R}\setminus\{-2,3\}$

$x=3$ V.A. $x=-2$ HOLE

$\frac{(x-1)(x-5)(x+2)}{(x-3)(x+2)}$

$\frac{x^3}{x^2}$ Slant asymptote

LONG DIVISION

No H.A.

$\pm 1, \pm 2, \pm 5, \pm 10$

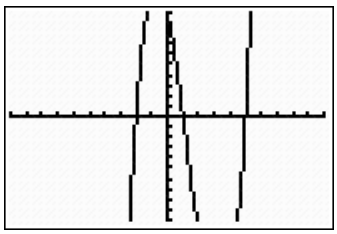
$(x-1)(x^2-3x-10)$

$(x-1)(x-5)(x+2)$

$1 \mid 1 \quad -4 \quad -7 \quad 10$

$\quad\quad 1 \quad -3 \quad -10$

$\quad 1 \quad -3 \quad -10$



x-int: $(1,0), (5,0)$

HOLE: $\frac{(-2-1)(-2-5)}{-2-3} = \frac{21}{-5}$

HOLE is $(-2, -\frac{21}{5})$

y-int: $(0, -\frac{10}{6}) = (0, -\frac{5}{3})$

Slant Asymptote: Long Division:

$\frac{x^3}{x^2} = x$

$\frac{-3x^2}{x^2} = -3$

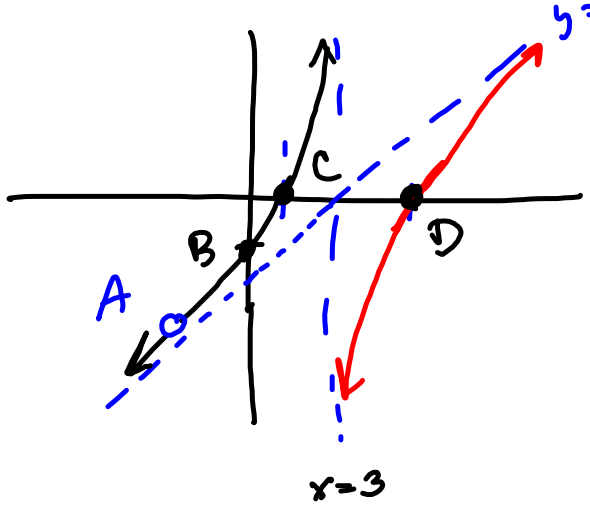
$x^2 - x - 6 \;\overline{)\; x^3 - 4x^2 - 7x + 10}$

$x - 3$

$-(x^3 - x^2 - 6x)$

$-3x^2 - x + 10$

$y = x - 3$ is slant asymptote!



$A = (-2, -\frac{21}{5})$

$B = (0, -\frac{5}{2})$

$C = (1, 0)$

$D = (5, 0)$

$\frac{1}{(x+2)(x+2)}$

$\frac{(x-5)(x-1)}{x-3} = \frac{x^2 - 6x + 5}{x-3}$

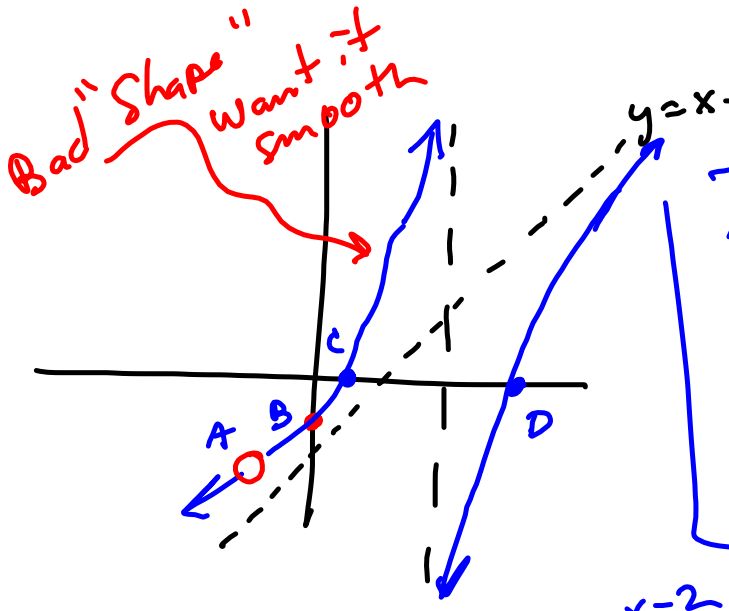
$3 \;\overline{)\; 1 \quad -6 \quad 5}$

$\quad\quad\;\; 3 \quad -9$

$\quad 1 \quad -3 \quad -4$

Slant Asymptote $y = x - 3$

y-int: $R^*(0) = \frac{10}{-6} = -\frac{5}{3}$ ⇝ $(0, -\frac{5}{3})$ = y-int

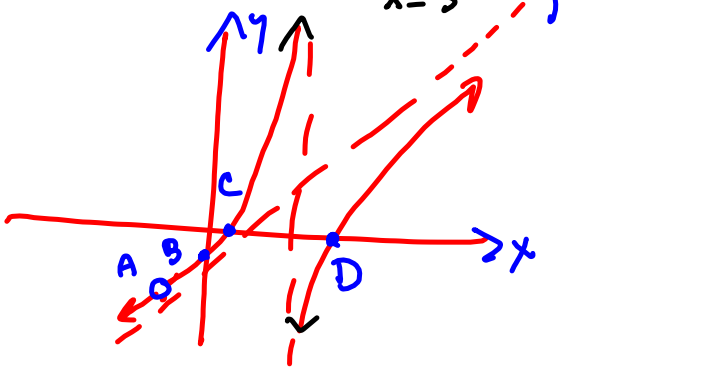


$A = (-2, -\frac{21}{5})$

$B = (0, -\frac{5}{3})$

$C = (1, 0)$

$D = (5, 0)$



§3.3 #75

$x^6 - x^5 - x^4 + x^3 - 12x^2 + 12x = 0$ Pull out common

$x(x^5 - x^4 - x^3 + x^2 - 12x + 12) = 0$

$a_5 = 1$ $a_0 = 12$ possible

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ rational zeros.

Descartes: $x^5 - x^4 - x^3 + x^2 - 12x + 12 = f(x)$

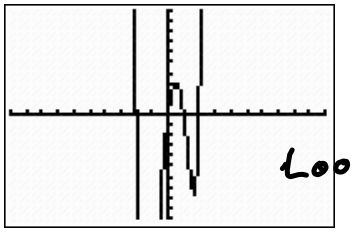
1 2 3 4

4, 2 or 0 positive

$f(-x) = -x^5 - x^4 + x^3 - x^2 + 12x + 12$

1

1 negative zero



Bounds on real zeros: Meh.

Looks like $x = -2, 0, 1, 2$, from graph.

$x(x+2)(x^4 - 3x^3 + 5x^2 - 9x + 6)$

-2 | 1 −1 −1 1 −12 12

−2 6 −10 18 −12

-2 | 1 −3 5 −9 6 0

−2 10 −30 No way.

1 −5 15 −39

$x(x+2)(x-1)(x-2)(x^2+3)$

$x^2 + 3 = 0$

(No real solns

$x^2 = -3$

$x = \pm i\sqrt{3}$

1 | 1 −3 5 −9 6

1 −2 3 −6

2 | 1 −2 3 −6 0

2 0 6

1 0 3 0

$x^2 + 3 = 0$

$a = 1, b = 0, c = 3$

$b^2 - 4ac = 0^2 - 4(1)(3)$

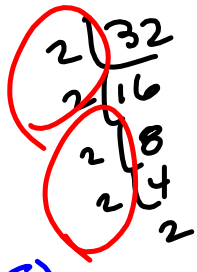
$= -12$

$x = \frac{\pm\sqrt{-12}}{2} = \frac{\pm i2\sqrt{3}}{2}$

$= \pm i\sqrt{3}$

$x(x+2)(x-1)(x-2)(x - i\sqrt{3})(x + i\sqrt{3})$

$$\frac{12 \pm \sqrt{-32}}{8} =$$



$$= \frac{12 \pm 4i\sqrt{2}}{8} = \frac{4(3 \pm i\sqrt{2})}{8} = \frac{3 \pm i\sqrt{2}}{2}$$

$$\frac{3x^2-5x+2}{11x^2+2} \xrightarrow{x \to BIG} \frac{3}{11} = y$$

$y = \frac{3}{11}$ is horizontal asymptote