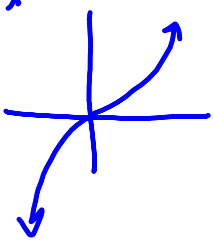
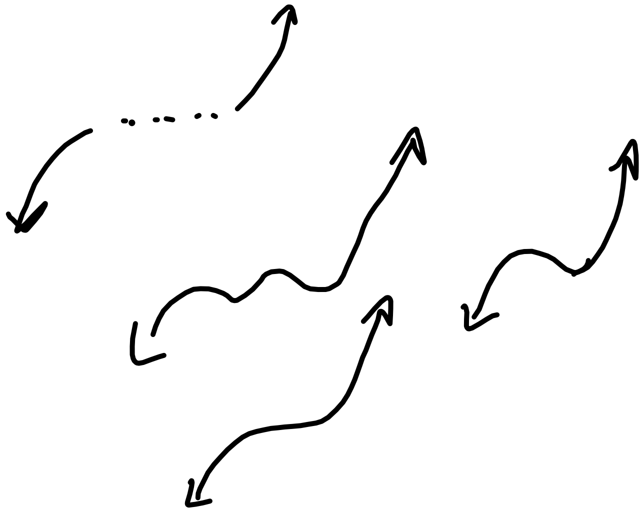
$$f(x) = x^5 - x^4 - 5x^3 + 9x^2 - 16x - 60$$

① End Behavior:

x^5 controls

x^5





② Descartes:

$$f(x) = \underbrace{x^5 - x^4}_{1} \underbrace{- 5x^3 + 9x^2}_{2} \underbrace{+ 9x^2 - 16x}_{3} - 60$$

$\boxed{3 \text{ or } 1}$ positive zeros.

$$f(-x) = -x^5 \underbrace{- x^4 + 5x^3}_{1} + 9x \underbrace{+ 16x - 60}_{2}$$

2 or 0 negative zeros

(3) Rational zeros

$(2x+3)(5x-7)=0 \Rightarrow x = -\frac{3}{2}, \frac{7}{5}$

$= 10x^2+x-21$

$a_n = a_2 = 10$

$a_0 = -21$

$\frac{p}{q}$: $\frac{\text{factors of } 21}{\text{factors of } 10}$

NOTE!

3 is a factor of 21

2 is " " " 10

$10x^2+x-21$

Possible Rational zeros:

$\frac{p}{q}$: $\pm 1, \pm 3, \pm 7, \pm 21$

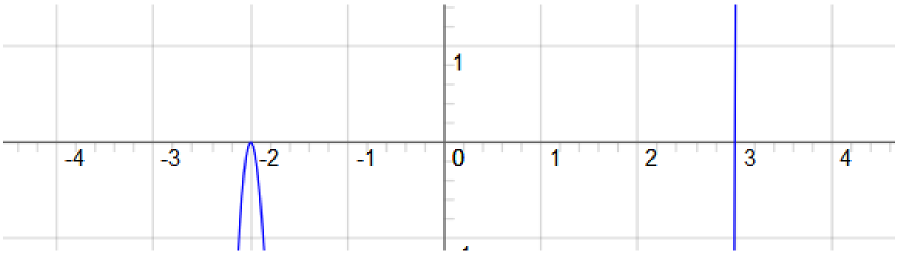
$\pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{7}{2}, \pm \frac{21}{2}$

$\pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{7}{5}, \pm \frac{21}{5}$

$\pm \frac{1}{10}, \pm \frac{3}{10}, \pm \frac{7}{10}, \pm \frac{21}{10}$

(3) $f(x) = x^5 - x^4 - 5x^3 + 9x^2 - 16x - 60$

$\frac{p}{q}$: $\frac{60}{1}$: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12,$
$\pm 15, \pm 20, \pm 30, \pm 60$



touches, so thinking repeated root.
(multiplicity = m = 2 or 4 ?)

$f(x) = x^5 - x^4 - 5x^3 + 9x^2 - 16x - 60$

Try $x = -2$ we split off a factor of $x+2$.

$f(x) = (x+2)(x^4 - 3x^3 + x^2 - 7x - 30)$

-2	1	-1	-5	9	-16	-60	
		-2	6	-2	-14	60	
-2	1	-3	1	7	-30	0	sweet!
		-2	10	-22	30		
3	1	-5	11	-15	0		sweet!
		3	-6	15			
	1	-2	5	0			sweet!

$(x+2)^2(x^3 - 5x^2 + 11x - 15)$

Depressed Polynomial.

$\boxed{(x+2)^2(x-3)(x^2-2x+5)}$

ANS to #5

$a = 1, b = -2, c = 5$

$b^2 - 4ac = (-2)^2 - 4(1)(5) = 4 - 20 = -16$

No real zeros. So $x^2 - 2x + 5$ is an irreducible quadratic factor

over the reals.

$$x = \frac{2 \pm \sqrt{-16}}{2(1)} = \frac{2 \pm i\sqrt{16}}{2} = \frac{2 \pm 4i}{2} = \frac{1 \pm 2i}{1}$$

$$= \frac{2(1 \pm 2i)}{2} = 1 \pm 2i$$

Here's where we are:

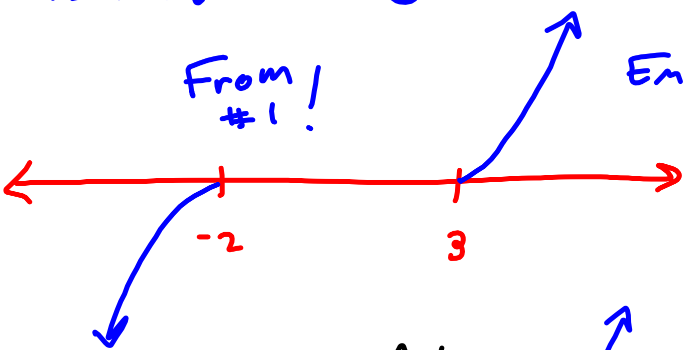
Ans to #6

$$\boxed{f(x) = (x+2)^2(x-3)(x-(1+2i))(x-(1-2i))}$$

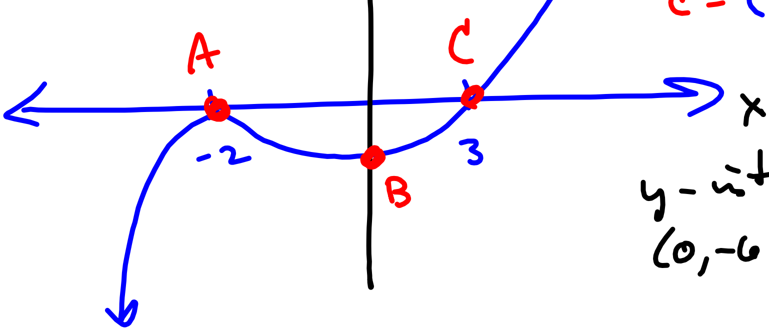
is split into linear factors

$$\left.\begin{array}{ll} x=-2 & m=2 \\ x=3 & m=1 \\ x=1+2i & m=1 \\ x=1-2i & m=1 \end{array}\right\} n=5$$

End Behavior



$(x+2)^2(x-3)(x^2-2x+5)$

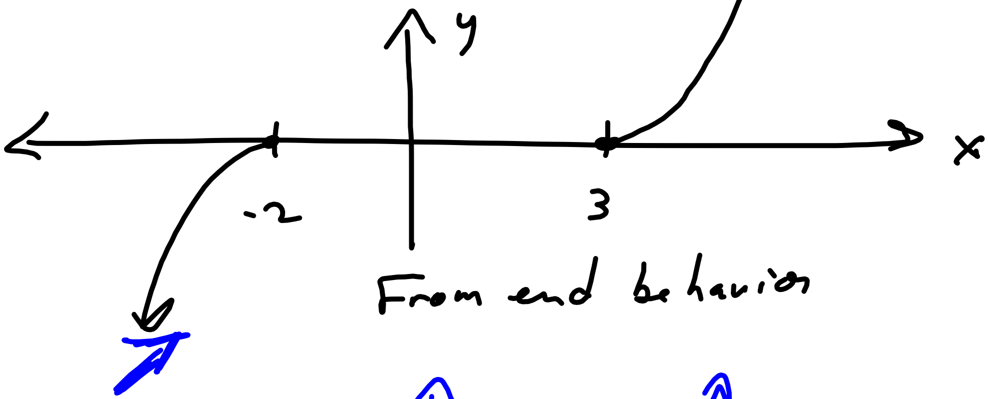


A = (-2,0)
B = (0,-60)
C = (3,0)

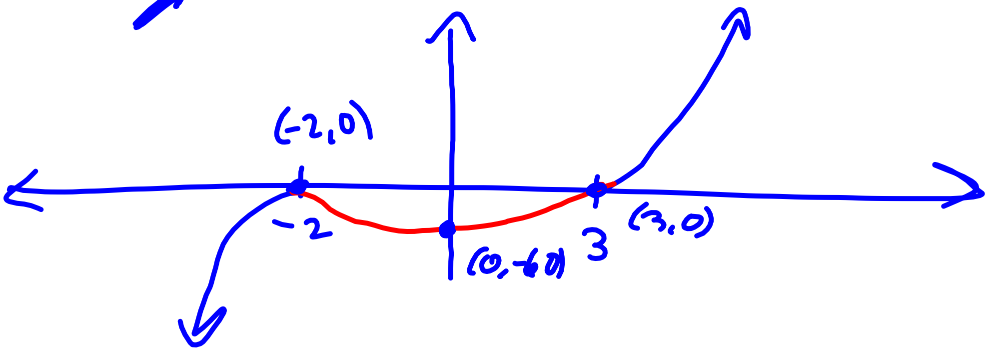
y-int:
(0,-60)

$x^5 + \cdots - 60$

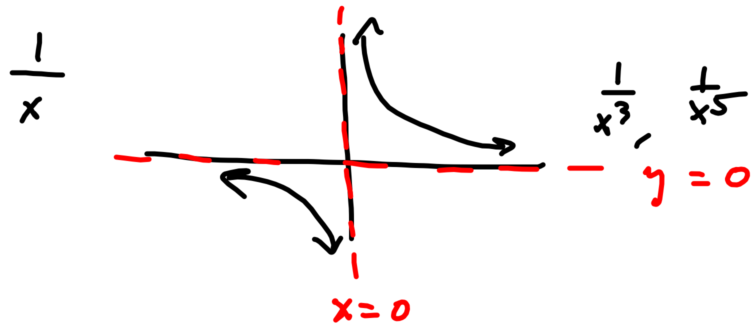
$(x+2)^2(x-3)(x^2-2x+5)$

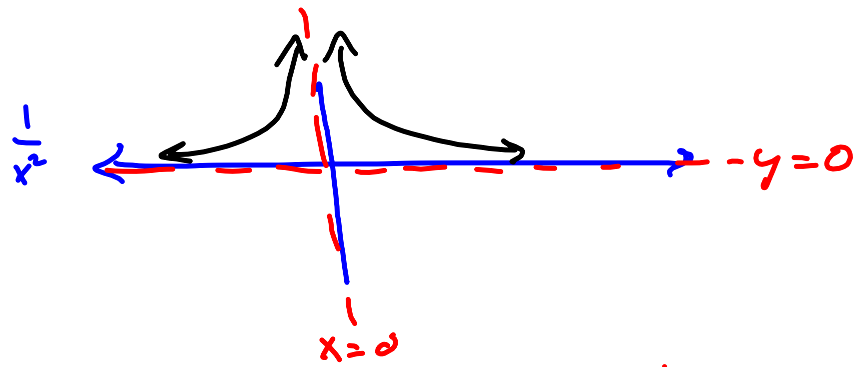


From end behavior



Analyzing the sign pattern:





Both are PROPER : $\dfrac{\text{Lower degree}}{\text{Higher degree}}$

$y = 0$ is horizontal asymptote.
For proper.

Impropers $\frac{\text{same degree}}{\text{same degree}}$

Horizontal Asymptote

$$y = \frac{\text{Big stuff}}{\text{Big stuff}}$$

$$\frac{3x^2+5x-6}{2x^2-17x+1000\pi} \xrightarrow{|x|\to\infty} \frac{3x^2}{2x^2} = \frac{3}{2} = y$$

$\frac{\text{Higher Degree}}{\text{Lower Degree}}$ Oblique/Slant Asymptote

$$R(x) = \frac{3x^3+2x-1}{x^2-5x+2}$$

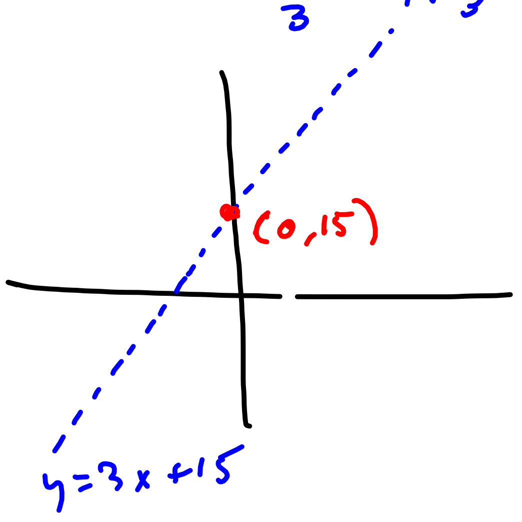
$$\begin{array}{r|l} & 3x+15 \quad r\ 71x-31 \\ \hline x^2-5x+2 & 3x^3+0x^2+2x-1 \\ & -(3x^3-15x^2+6x) \\ \hline & 15x^2-4x-1 \\ & -(15x^2-75x+30) \\ \hline & 71x-31 \end{array}$$

So, $R(x) = \frac{3x^3+2x-1}{x^2-5x+1} = \boxed{3x+15} + \frac{71x-31}{x^2-5x+2}$

So, its end behavior is like $3x+15$

$R(x)$ approaches $y=3x+15$ off to left & off to right.

$$\frac{28}{3} = 9+\frac{1}{3}$$



$$R(x) = \frac{2x^2 - x - 3}{x^2 + 2x - 15} = \frac{(2x-3)(x+1)}{(x+5)(x-3)}$$

Domain: $x \neq -5, 3$

No cancels, so

$\boxed{x = -5,\ x = 3}$ Vertical Asymptotes

V.A.

Zeros: $x = -1, \frac{3}{2}$

x-ints: $(-1, 0), (\frac{3}{2}, 0)$

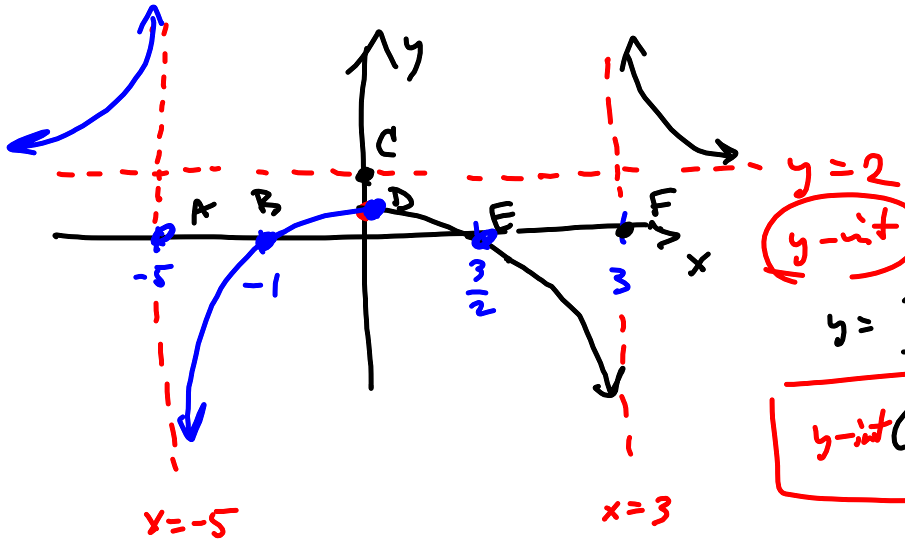
$-5 \quad -1 \quad \frac{3}{2} \quad 3$

$=0 \quad =0$

H.A.: $x \to \pm\infty$

Top deg = Bottom Deg, so...

$\frac{2x^2}{x^2} = \boxed{2 = y}$ is Horizontal Asymptote,



y-int:

$y = \frac{-3}{-15} = \frac{1}{5} = y$

$\boxed{\text{y-int } (0, \frac{1}{5})}$

$A = (-5, 0)$ $E = (\frac{3}{2}, 0)$

$B = (-1, 0)$ $F = (3, 0)$

$C = (0, 2)$

$D = (0, \frac{1}{5})$

$\frac{1}{x-3}$ changes sign @ $x = 3$