Recall $x^2+5x+6=0$ when $x=-2$, $x=-3$

$(x+2)(x+3)=0$

$a=1, b=5, c=6$

$b^2-4ac = 5^2-4(1)(6) = 25-24 = 1 = 1^2$

BLEAH!

$-5 \pm \sqrt{5^2-4(1)(6)}$

$x = \frac{-5\pm\sqrt{1}}{2} = \frac{-5\pm 1}{2}$ $\nearrow \frac{-4}{2}=-2$ $\searrow \frac{-6}{2}=-3$

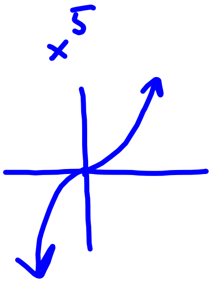
$x=-2$ $x=-3$

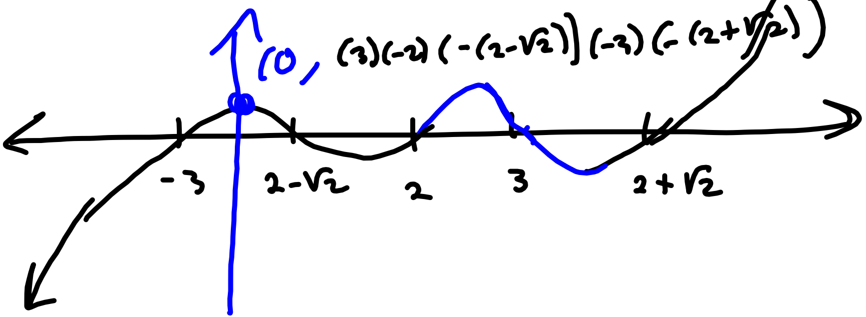
$(x+2)$ $(x+3)$

Build a polynomial in factored form with zeros $x = -3, 2-\sqrt{2}, 2, 3, 2+\sqrt{2}$

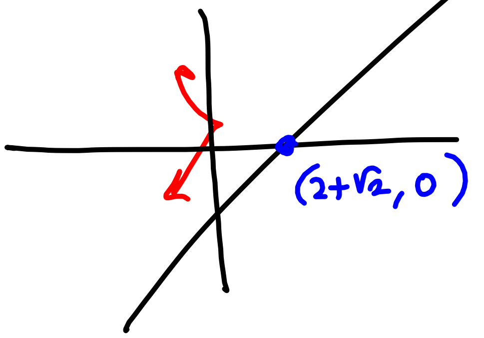
$\underline{(x-(-3))(x-2)(x-(2-\sqrt{2}))(x-3)}(x-(2+\sqrt{2}))$

$= x^5 +$ smaller stuff





$y = x-(2+\sqrt{2})$



$f(x)$ has zeros of $2+i, 2-i$

Factored form is $f(x) = (x-(2+i))(x-(2-i))$ STOP!

Never Expand unless specifically asked to expand.

Expand the above:

$$(x-2-i)(x-2+i)$$

$$= x^2 - 2x \boxed{+ix} - 2x + 4 \underline{-2i} \boxed{-ix} \underline{+2i} - i^2$$

$$= x^2 - 4x + 4 - (-1) = x^2 - 4x + 5$$

↗

Consider $P(x) = x^3 - 27 = x^3 - 3^3$

$$= (x-3)(x^2+3x+9)$$

$$x^3 - 27 = 0$$
$$x^3 = 27$$
$$\sqrt[3]{x^3} = \sqrt[3]{27}$$
$$x = 3$$

That says $x=3$ is a zero

" " $x-3$ is a factor.

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$
$$a^3 + b^3 = (a+b)(a^2-ab+b^2)$$

We can capture the x^2+3x+9 by synthetic division.

Divide x^3-27 by $x-3$

I'm gonna split off a factor of $x-3$.

$$\begin{array}{r|rrrr} 3 & 1 & 0 & 0 & -27 \\ & & 3 & 9 & 27 \\ \hline & 1 & 3 & 9 & 0 \\ & x^2 & x^1 & c & r \end{array}$$

Sweet!

This says $x^3-27 = \boxed{(x-3)\underbrace{(x^2+3x+9)}_{\text{Depressed Polynomial}}}$

Once we found
$x=3$, did a division,
we turned $x^3-27=0$ into
$(x-3)(x^2+3x+9)=0$ &
we can clobber $x^2+3x+9=0$
& split x^3-27 into linear factors.

$a=1,\ b=3,\ c=9$

$b^2-4ac = 3^2-4(1)(9) = 9-36 = -27$

$\sqrt{-27}$
$= 3i\sqrt{3}$

$$x = \frac{-3 \pm 3i\sqrt{3}}{2}$$

3 | 27
3 | 9
3

$\sqrt{27} = 3\sqrt{3}$

Factored form

$$\underline{(x-3)}^1\left(x-\left(\underline{\frac{-3+3i\sqrt{3}}{2}}\right)\right)^1\left(\underline{x-\left(\frac{-3-3i\sqrt{3}}{2}\right)}\right)^1$$

↗

Divide $f(x) = 3x^5 - 4x^2 + 11x - 20$

by $x + 2 = 0$ when $x = -2$, so,

$$\begin{array}{r|rrrrrr} -2 & 3 & 0 & 0 & -4 & 11 & -20 \\ & & -6 & 12 & -24 & 56 & -134 \\ \hline & 3 & -6 & 12 & -28 & 67 & \boxed{-154} \\ & x^4 & x^3 & x^2 & x & c & r \end{array}$$

$$3x^5 - 4x^2 + 11x - 20 = (x+2)(3x^4 - 6x^3 + 12x^2 - 28x + 67) - 154$$

What's $f(-2)$? Remainder Theorem!

FACTOR THEOREM is just the REMAINDER Theorem when Remainder is 0.

$x = 2, -3, 7, 9$

$x^4 - 15x^3 + 41x^2 + 159x - 378$

2	1	-15	41	159	-378	
		2	-26	30	378	
-3	1	-13	15	189	0	Sweet!
		-3	48	-189		
	1	-16	63	0	Sweet!	

$$(x-2)\underbrace{(x^3-13x^2+15x+189)}_{\text{Depressed polynomial}}$$

$$(x-2)(x+3)(x^2-16x+63)$$

$$x^2-16x+63 = (x-9)(x-7)$$

That means

$$f(x) = (x-2)(x+3)(x-9)(x-7)$$