$f(x) = \dfrac{x^3 + \pi x - e}{x-5}$, $g(x) = \sqrt{x+4}$

$\mathcal{D}$ = Domain. $\mathcal{D}(f)$: Need $x - 5 \neq 0$

$\{x \mid x \neq 5\} = \mathbb{R} \setminus \{5\}$

$= (-\infty, 5) \cup (5, \infty)$

$\mathcal{D}(g)$: Need $x + 4 \geq 0$

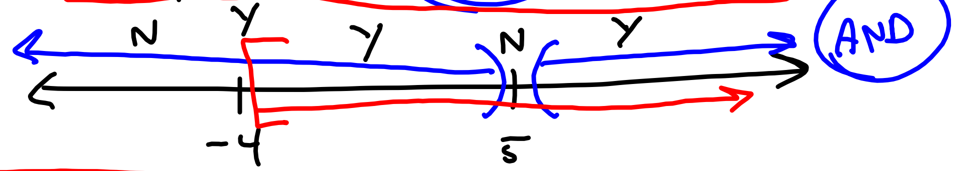
$\{x \mid x \geq -4\} = [-4, \infty)$

$(f+g)(x) = f(x) + g(x) = \dfrac{x^3 + \pi x - e}{x-5} + \sqrt{x+4}$

$\mathcal{D}(f+g) = \mathcal{D}(f) \cap \mathcal{D}(g)$

$= \{x \mid x \in \mathcal{D}(f) \text{ AND } x \in \mathcal{D}(g)\}$

$= \boxed{\{x \mid x \neq 5 \text{ AND } x \geq -4\}}$

= 

$= \boxed{[-4, 5) \cup (5, \infty)}$

$= \mathcal{D}(f-g) = \mathcal{D}(fg)$, but

$\mathcal{D}\left(\frac{f}{g}\right)$ adds one wrinkle:

$g(x) \neq 0$

$\sqrt{x+4} \neq 0$

$(\sqrt{x+4})^2 \neq 0^2$

$x + 4 \neq 0$

$x \neq -4$

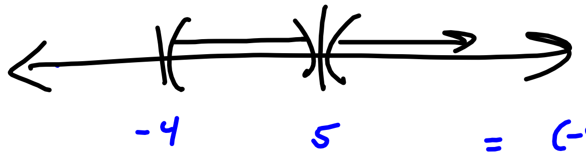
$A = B$

$A^2 = B^2$

$\dfrac{f}{g} = \dfrac{\frac{x^3 + \pi x - e}{x-5}}{\sqrt{x+4}}$

$= \dfrac{x^3 + \pi x - e}{(\sqrt{x+4})(x-5)}$

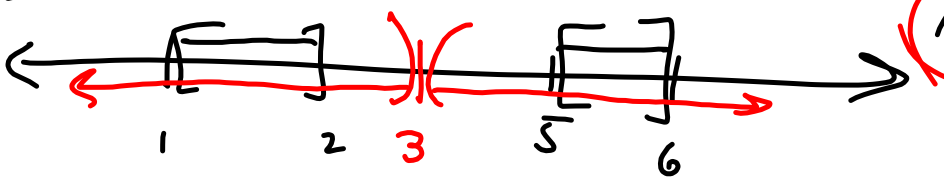
$\mathcal{D}(f/g) = \{x \mid x \neq 5 \text{ and } \underbrace{x \geq -4 \text{ and } x \neq -4}_{x > -4}\}$



$= (-4, 5) \cup (5, \infty)$

$$\Big((-\infty,3)\cup(3,\infty)\Big)\cap\Big([1,2]\cup[5,6]\Big)$$

AND



$$[1,2]\cup[5,6]$$

$x \in$ OR x then $x \in$ BOTH RED & BLACK.

$(f \circ g)(x) = f(g(x))$ g: $\sqrt{x+4}$, f: $\frac{\text{STUFF}}{x-5}$

* g has to eat x & f has to eat g(x) *

$$\boxed{\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}}$$

$$= \{x \mid x \geq -4 \text{ and } \sqrt{x+4} \neq 5\}$$

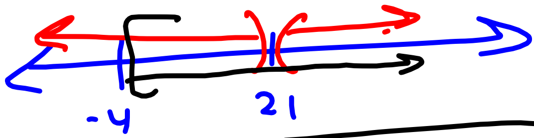
Scratch: $\sqrt{x+4} \neq 5$

$(\sqrt{x+4})^2 \neq 5^2$

$x+4 \neq 25$

$x \neq 21$

$\{x \mid x \geq -4 \text{ and } x \neq 21\}$



$$\boxed{\begin{aligned} &= [-4, 21) \cup (21, \infty) \\ &= \mathcal{D}(f \circ g) \end{aligned}}$$

$f(x) = \dfrac{x+2}{x-7}$ is 1-to-1.

Proof: TEST QUESTION !

Suppose $f(x_1) = f(x_2)$. Then $\dfrac{x_1+2}{x_1-7} = \dfrac{x_2+2}{x_2-7}$

cross-multiply :

$\Longrightarrow (x_1+2)(x_2-7) = (x_2+2)(x_1-7)$ Expand

$\Longrightarrow x_1x_2 - 7x_1 + 2x_2 - 14 = x_2x_1 - 7x_2 + 2x_1 - 14$

collect.

$\Longrightarrow -7x_1 + 2x_2 = -7x_2 + 2x_1$

$\Longrightarrow -9x_1 = -9x_2$

$\Longrightarrow x_1 = x_2$! So it IS 1-to-1! ∎

Recall :

f is 1-to-1 means :

$x_1 \neq x_2 \Longrightarrow f(x_1) \neq f(x_2)$

or, equivalently,

$f(x_1) = f(x_2) \Longrightarrow x_1 = x_2$ &

we used this last way in our proof.