Compound Interest

$$A = P\left(1+\frac{r}{m}\right)^{mt} = P(1+i)^n$$

$A = P(1+i)^n$ Given to you on cheatsheet

Annuity

$$FV = R\left(\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right)$$

$$FV = R\left(\frac{(1+i)^n-1}{i}\right)$$

Loans: Banker's gonna get his compound interest by buying an annuity from YOU!

§4.4 II

Comparing investments #91

Fiona: 1000 @ 6% compdd cntnsly

Maria: 1100 @ 6% compounded daily

How long, to nearest day, until they have same amt of $?

$1000e^{.06t} = 11000\left(1+\frac{.06}{360}\right)^{360t}$ Solve for t.

* Banker's year is typically 360 days, by convention.

$\ln\left(1000e^{.06t}\right) = \ln\left(1100\left(1+\frac{.06}{360}\right)^{360t}\right)$

$\ln(1000) + \ln(e^{.06t}) = \ln(1100) + \ln\left(\left(1+\frac{.06}{360}\right)^{360t}\right)$

$a + .06t = b + (360t)\ln\left(1+\frac{.06}{360}\right)$

$a + .06t = b + (360t)c$

$.06t - 360ct = b - a$

$(.06 - 360c)t = b - a$

$t = \frac{b-a}{.06-360c} = \frac{\ln(1100)-\ln(1000)}{.06-360\left(\ln\left(1+\frac{.06}{360}\right)\right)}$

$\approx \frac{.09531\ 01798}{.06 - 360(.06)}$

```
1/.4
                  2.5
ln(2.5)/ln(1.007
5)
      122.6296724
ln(1100/1000)
       .0953101798
ln(2.5)/ln(1.007
5)
      122.6296724
ln(1100/1000)
       .0953101798
360*(1+.06/360)
             360.06
ln(1100/1000)/
```

$1000e^{.06t} = 11000\left(1+\frac{.06}{360}\right)^{360t}$

$= 1100\left(1.000\overline{16}\right)^{360t}$

$e^{.06t} = \frac{1100}{1000}\left(1.000\overline{16}\right)^{360t}$

$e^{.06t} = 1.1\left(1.000\overline{16}\right)^{360t}$

$\ln(ab) = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\frac{\ln a}{\ln b}$ doesn't simplify!

$\ln(\quad) = \ln(\quad)$:

$.06t = \ln\left(1.1\left(1.000\overline{16}\right)^{360t}\right)$

$.06t = \ln(1.1) + \ln\left(\left(1.000\overline{16}\right)^{360t}\right)$

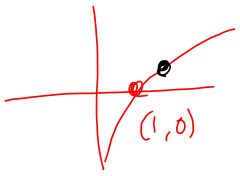
$= \ln(1.1) + \left(\ln\left(1.000\overline{16}\right)\right)(360t)$

$.06t - (360t)\ln\left(1.000\overline{16}\right) = \ln(1.1)$

$\left(.06 - 360\ln\left(1.000\overline{16}\right)\right)t = \ln(1.1)$

(left: negative; right: positive)

$\Rightarrow t$ is negative, so I'm messing up.



$1000\, e^{0.06\,t} = 1100\ 1.000166667^{360\,t} \sim 19532.91793$

$$1000\, e^{0.06\,t} = 1100\ 1.000166667^{360\,t}$$

$\Rightarrow t \approx 19532.91793$ by magic.

$1000\, e^{.06t} = 1100\left(1+\frac{.06}{360}\right)^{360t}$

$\ln 1000 + \ln(e^{.06t}) = \ln(1100) + \ln\left(\left(1+\frac{.06}{360}\right)^{360t}\right)$

$\ln 1000 + .06t = \ln 1100 + (360t)\ln\left(1+\frac{.06}{360}\right)$

$a + .06t = b + 360ct$ (where $c = \ln\left(1+\frac{.06}{360}\right)$)

$.06t - 360ct = b - a$

$t(.06 - 360c) = b - a = \ln 1100 - \ln 1000 = \ln\left(\frac{1100}{1000}\right)$

Backwards $t = \dfrac{b-a}{.06-360c} = \dfrac{\ln(1.1)}{.06 - 360\left(\ln\left(1+\frac{.06}{360}\right)\right)}$

```
ln(1000/1100)
     -.0953101798
Ans/(.06-360*ln(
1+.06/360))
     -19064.19968
```

```
1+.06/360))
     -19064.19968
ln(1100/1000)
      .0953101798
Ans/(.06-360*ln(
1+.06/360))
      19064.19968
```

Still failing to get this to come out right.