$$x + 2y - 3z = -17$$
$$3x - 2y - z = -3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & a & b \\ 0 & 1 & c & d \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -17 \\ 3 & -2 & -1 & -3 \end{array}\right]$$

Scratch:

$$\begin{array}{lcccc} -3R1 & -3 & -6 & 9 & 51 \\ R2 & 3 & -2 & -1 & -3 \\ \hline & 0 & -8 & 8 & 48 \end{array}$$

$$\begin{array}{lcccc} -\frac{1}{8}R2 & 0 & 1 & -1 & -6 \end{array}$$

$$\begin{array}{lcccc} -2R2 & 0 & -2 & 2 & 12 \\ R1 & 1 & 2 & -3 & -17 \\ \hline & 1 & 0 & -1 & -5 \end{array}$$

$$\begin{array}{l} R1 \\ -3R1+R2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -3 & -17 \\ 0 & -8 & 8 & 48 \end{array}\right]$$

$$\begin{array}{l} R1 \\ -\frac{1}{8}R2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -3 & -17 \\ 0 & 1 & -1 & -6 \end{array}\right]$$

$$\begin{array}{l} -2R2+R1 \\ R2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -5 \\ 0 & 1 & -1 & -6 \end{array}\right]$$

z is free!

So our solution set has one degree of freedom.

It's a line!

INTERPRET

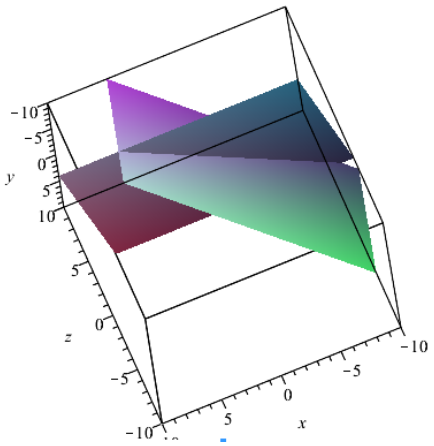
$$x - z = -5$$
$$y - z = 6$$

$$x = z - 5$$
$$y = z - 6$$
$$z = z!$$

$$\{(x, y, z) \mid x = z-5,\ y = z-6,\ z \in \mathbb{R}\}$$

$$x = z - 5$$
$$y = z - 6$$

$$\begin{bmatrix} z-5 \\ z-6 \\ z \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$$



$$x + az = b$$
$$y + cz = d$$

$$x = -az + b$$
$$y = -cz + d$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -az+b \\ -cz+d \\ z \end{bmatrix} = \begin{bmatrix} -ar+b \\ -cr+d \\ r \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 5 \\ 2 & -4 & 6 & 3 \\ 2 & -3 & 1 & 9 \end{array}\right] \begin{array}{l} R1 \\ -2R1+R2 \\ -2R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 5 \\ 0 & 0 & 0 & -7 \\ 0 & & & \end{array}\right]$$

$0 = -7$?!

No!

No soln!

$$\begin{bmatrix} 1 & 1 & 2 & 7.5 \\ 3 & 4 & 1 & 12 \\ 5 & 2 & 5 & 21 \end{bmatrix} \xrightarrow{\substack{-3R1+R2 \\ -5R1+R3}} \begin{bmatrix} 1 & 1 & 2 & 7.5 \\ 0 & 1 & -5 & -10.5 \\ 0 & -3 & -5 & -16.5 \end{bmatrix}$$

$$\begin{array}{r} 7.5 \\ \times\ -3 \\ \hline -22.5 \end{array} \qquad \begin{array}{r} -22.5 \\ 12 \\ \hline -10.5 \end{array}$$

$$\begin{array}{r} 7.5 \\ \times\ -5 \\ \hline -37.5 \end{array} \qquad \begin{array}{r} -37.5 \\ 21 \\ \hline -16.5 \end{array}$$

$$3R2+R3 \begin{bmatrix} 1 & 1 & 2 & 7.5 \\ 0 & 1 & -5 & -10.5 \\ 0 & 0 & -20 & -48. \end{bmatrix}$$

$$\begin{array}{r} -31.5 \\ -16.5 \\ \hline -48.0 \end{array}$$

$$-\tfrac{1}{10}R3 \begin{bmatrix} 1 & 1 & 2 & 7.5 \\ 0 & 1 & -5 & -10.5 \\ 0 & 0 & 1 & -4.8 \end{bmatrix}$$

$z = -4.8$

$y - 5z = -10.5$

$y = 5(-4.8) - 10.5$

$= -24 - 10.5 = -34.5 = y$

$x + -34.5 + 2(-4.8) = 7.5$

$x - 24.5 - 9.6 = 7.5$

$x - 44.1 = 7.5$

$x = 44.1 + 7.5$

$x = 51.6$

+2.4

All this is based on bad info

Row-reduction steps with Tech:

$$\begin{bmatrix} 1 & 1 & 2 & 7.5 \\ 0 & 1 & -5 & -10.5 \\ 5 & 2 & 5 & 21 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 & 7.5 \\ 0 & 1 & -5 & -10.5 \\ 0 & -3 & -5 & -16.5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 & 7.5 \\ 0 & 1 & -5 & -10.5 \\ 0 & 0 & -20 & -48.0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 7.5 \\ 0 & 1 & -5 & -10.5 \\ 0 & 0 & 1 & 2.400000000 \end{bmatrix}$$

Echelon Form / upper Triangular Form. We back-sub, using $z = 2.4$ to get x & y.

Gauss-Jordan takes us to Reduced-row-echelon form

$$\begin{bmatrix} 1. & 0. & 0. & 1.20000000000000 \\ 0. & 1. & 0. & 1.50000000000000 \\ 0. & 0. & 1. & 2.40000000000000 \end{bmatrix}$$

and we can read $x = 1.2, y = 1.5, z = 2.4$ directly