Compound Interest:

Savings Account: $A(t) = P\left(1+\frac{r}{m}\right)^{mt} = P(1+i)^n$

Annuity (Pmts @ end of month)

Future value $= FV = R\left(\frac{(1+i)^n - 1}{i}\right) = R\left(\frac{\left(1+\frac{r}{m}\right)^{mt} - 1}{\left(\frac{r}{m}\right)}\right)$

Calculate Loan Pmts: When you know what you borrowed.

$A(t) = FV$

"AMORTIZATION" $P(1+i)^n = R\left(\frac{(1+i)^n - 1}{i}\right)$ Solve for R.

$$R = P(1+i)^n\left(\frac{i}{(1+i)^n - 1}\right)$$

$$= \frac{Pi}{(1+i)^{-n}\left((1+i)^n - 1\right)} = \frac{Pi}{1 - (1+i)^{-n}} = Pmt = R$$

where P is (known) amt borrowed

Sinking Fund: You know the future value, and want to know the pmt.

(Setting aside for kids' college, Firm setting aside $ for possible court damages or new equipment)

FV known, R is not.

$R\left(\frac{(1+i)^n - 1}{i}\right) = FV$ = amt you want in the bank at the end:

$$R = \frac{(FV)\,i}{(1+i)^n - 1}$$

"Sinking Fund" How much do I set aside each month if I want $100,000 in the kids' college fund in 18 yrs.

E Savings: What's Amt in bank if you deposit $500 @ 5% Apr for 10 yrs, if interest is compounded daily?

$m = 365$

$r = .05$

$P = \$500$

$t = 10$

$i = \frac{r}{m} = \frac{.05}{365}$

$n = mt = (365)(10)$

$A = P\left(1+\frac{r}{m}\right)^{mt} = 500\left(1+\frac{.05}{365}\right)^{(360)(10)}$

Book uses 360-day year. Difference is pennies.

```
      824.3606354
500(1+.05/365)^(
365*10)
       824.332407
500(1+.05/360)^(
360*10)
      824.3320149
```

→ 365-day year

→ 360-day year

```
365*10)
       824.332407
500(1+.05/360)^(
360*10)
      824.3320149
500e^(.05*10)
      824.3606354
```

→ Continuous compounding

$A = Pe^{rt}$

Loan Payments of $500/month @ 7% Apr, compounded monthly for 5 years.
How much money was borrowed?

Present Value of the annuity.

$$P(1+i)^n = R\left(\frac{(1+i)^n - 1}{i}\right)$$ Solve for P!

$$P = PV = R\left(\frac{1-(1+i)^{-n}}{i}\right)$$ from

$$R\left(\frac{(1+i)^n-1}{i}\right)\left(\frac{1}{(1+i)^n}\right) = R\left(\frac{(1+i)^n-1}{i}\right)\left(\frac{(1+i)^{-n}}{1}\right)$$

$$= R\left(\frac{(1+i)^n(1+i)^{-n} - 1\,(1+i)^{-n}}{i}\right) = R\left(\frac{1-(1+i)^{-n}}{i}\right)$$

→ Also from this eqn is Loan Pmt when you know how much you borrowed. (Solve for R)

$$R = \left(P(1+i)^n\right)\left(\frac{i}{(1+i)^n-1}\right) = \frac{P}{(1+i)^{-n}}\left(\frac{i}{(1+i)^n-1}\right)$$

$$\text{Pmt} = R = \frac{Pi}{1-(1+i)^{-n}}$$

Summary

$A = P\left(1+\frac{r}{m}\right)^{mt}$ Compound Interest

Future value of an annuity $FV = R\left(\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\left(\frac{r}{m}\right)}\right)$

Sinking Fund FV Known How much are pmts?

$R = \frac{(FV)\,i}{(1+i)^n - 1}$

Amortization (Loan Pmt)

PRESENT value is known

Pmt $= R =$ (

Advise NOT doing a separate calculation, but just enter the expression. If you can't do it in one expression, like I do, then DON'T ROUND OFF until the final answer!

Amortization Schedule

Period	Balance	Interest
1	$B_1 = B_0 - R + B_0 i$	$B_0 i$
2	$B_2 = B_1 - R + B_1 i$	$B_1 i$
3	$B_3 = B_2 - R + B_2 i$	$B_2 i$
⋮		

Amt Borrowed $= B = B_0$

Balance at end of Period n is B_n

$R =$ pmt

$i = \frac{r}{m}$ = interest rate per period.