

$$g(x) = 2 \cdot 3^{-2x+10} - 11 = y$$

Find $g^{-1}(x)$: Swap x & y .

$$2 \cdot 3^{-2y+10} - 11 = x$$

Solve for y .

$$\left. \begin{aligned} 2 \cdot 3^{-2y+10} &= x+11 \\ 3^{-2y+10} &= \frac{x+11}{2} \end{aligned} \right\}$$

Isolate 3 stuff ADD 11
Divide by 2

$$\log_3(3^{-2y+10}) = \log_3\left(\frac{x+11}{2}\right)$$

Extract stuff w/ $\log_3(*)$

$$-2y+10 = \log_3\left(\frac{x+11}{2}\right)$$

See LHS?
 $\log_3(*)$ brought that
"stuff" down to earth.

$$-2y = \log_3\left(\frac{x+11}{2}\right) - 10 \quad \text{Isolate } -2y \text{ by subtracting } 10$$

$$\boxed{y = \frac{\log_3\left(\frac{x+11}{2}\right) - 10}{-2} = g^{-1}(x)}$$

$$= f(x)$$

So, $f(x) = g^{-1}(x)$, if we did it right.

$$(f \circ g)(x) \stackrel{?}{=} x$$

$$f(g(x)) = \frac{\log_3\left(\frac{g(x)+11}{2}\right) - 10}{-2}$$

$$= \frac{\log_3\left(\frac{2 \cdot 3^{-2x+10} - 11 + 11}{2}\right) - 10}{-2} = \frac{\log_3(2 \cdot 3^{-2x+10}) - 10}{-2}$$

$$= \frac{\log_3(2) + \log_3(3^{-2x+10}) - 10}{-2}$$

$$2 \cdot 3^{-2y+10} - 11 = x$$

$$2 \cdot 3^{-2y+10} = x+11$$

$$3^{-2y+10} = \frac{x+11}{2}$$

$$\log_3(\ast) = \log_3\left(\frac{x+11}{2}\right)$$

$$-2y+10 = \log_3\left(\frac{x+11}{2}\right)$$

$$-2y = \log_3\left(\frac{x+11}{2}\right) - 10$$

$$y = \frac{\log_3\left(\frac{x+11}{2}\right) - 10}{-2} = g^{-1}(x) = f(x)$$

$$(f \circ g)(x) = f(g(x)) = \frac{\log_3\left(\frac{g(x)+11}{2}\right) - 10}{-2}$$

$$= \frac{\log_3\left(\frac{2 \cdot 3^{-2x+10} - 11 + 11}{2}\right) - 10}{-2} = \frac{\log_3\left(\frac{2 \cdot 3^{-2x+10}}{2}\right) - 10}{-2}$$

$$= \frac{\log_3\left(3^{-2x+10}\right) - 10}{-2} = \frac{-2x+10 - 10}{-2} = \frac{-2x}{-2} = x \quad !$$

Logs bring exponents
down to Earth.

Computers are adding machines.

To multiply $64 \cdot 32$, take \log_2 (*)

$$\log_2(y) = \log_2(64 \cdot 32) = \log_2(64) + \log_2(32)$$

$$= 6 + 5 = 11, \text{ so}$$

$$2^{\log_2(y)} = y = 2^{11} = 64 \cdot 32 \quad |$$