8. (3 pts) Sketch the graph of $R(x)=\dfrac{x^2-5x-6}{x^2-5x+6}$.

zeros upstairs : x-ints.

" downstairs: vertical asymptotes

Same zero upstairs & downstairs : HOLE.

$$R(x) = \frac{(x-6)(x+1)}{(x-3)(x-2)} \quad \begin{matrix} \longrightarrow x=6,\ x=-1 \\ \longrightarrow x=2,\ x=3 \end{matrix}$$

$\mathcal{D}: \mathbb{R} \setminus \{3,2\}$

$\boxed{\text{V.A.: } x=2,\ x=3}$

$\boxed{\text{x-int: } (6,0),\ (-1,0)}$

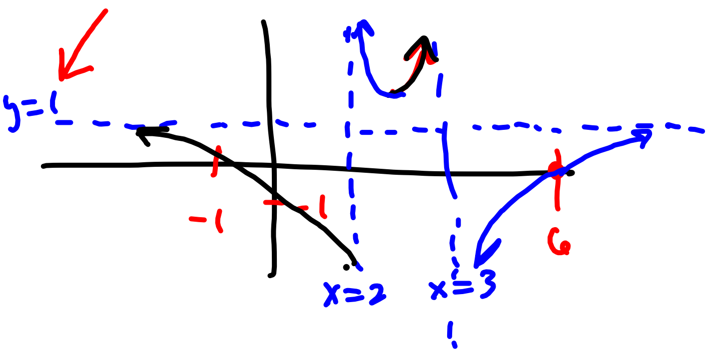
Horiz. Asymptote

$$\frac{x^2-5x-6}{x^2-5x+6} \xrightarrow{x \to \text{BIG}} \frac{x^2}{x^2} = 1$$

$\boxed{\text{H.A. } y=1}$

y-int:

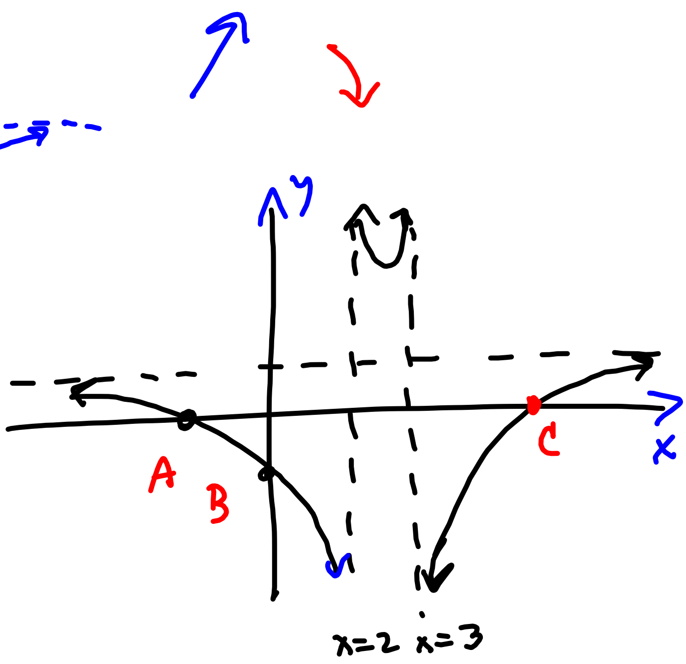
$$f(0) = \frac{x^2-5x-6}{x^2-5x+6} = \frac{-6}{6} = -1 \rightsquigarrow \boxed{(0,-1) \text{ y-int}}$$



$A=(-1,0)$

$B=(0,-1)$

$C=(6,0)$

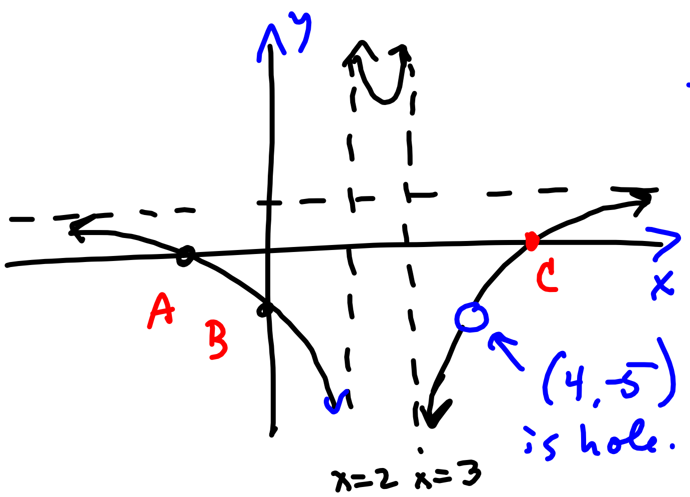


There's an $R(x)=\frac{x^2-5x-6}{x^2-5x+6} \doteq \frac{(x+1)(x-6)}{(x-3)(x-2)}$ Living inside!

We just find the hole!

9. (3 pts) The graph of $g(x)=\frac{x^3-9x^2+14x+24}{x^3-9x^2+26x-24}$ differs from the graph of f, in #8, in only one small detail.

Sketch the graph of g, showing all asymptotes, intercepts and holes.



Numerator:

$x^3-9x^2+14x+24$

$$\begin{array}{r|rrrr} -1 & 1 & -9 & 14 & 24 \\ & & -1 & 10 & -24 \\ \hline 6 & 1 & -10 & 24 & 0 \\ & & 6 & -24 & \\ \hline & 1 & -4 & 0 & \end{array}$$

So $x-4$ is the factor.

So, $g(x)=\frac{(x+1)(x-6)\cancel{(x-4)}}{(x-2)(x-3)\cancel{(x-4)}}$

$=R(x)$, when $x\neq 4$

Hole @ $x=4$

$R(4)=\frac{(4+1)(4-6)}{(4-2)(4-3)}$

$=\frac{(5)(-2)}{(2)(1)}=\frac{-10}{2}=-5$

$\boxed{\text{HOLE: } (4,-5)}$

END Behavior: Rational Func.

$\frac{\text{LOWER Degree}}{\text{HIGHER Degree}}$ = Proper : $y=0$ is H.A.

$\frac{\text{SAME}}{\text{SAME}}$ = IMPROPER. Just look @ highest degree terms

Previous prob. $\frac{x^2-5x-6}{x^2-5x+6} \xrightarrow{x \to BIG} \frac{x^2}{x^2} = 1 = y$

$\frac{\text{Higher Degree}}{\text{Lower Degree}}$ = Improper. Leads to slant asymptote we find by long division.

10. (3 pts) Sketch the graph of $R(x)=\frac{x^3-4x^2-7x+10}{x^2-x-6}$

Long Division

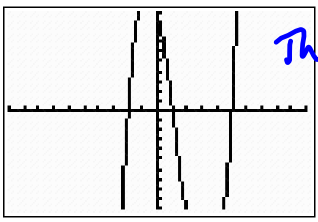
$\frac{x^3}{x^2} = x$

$$\begin{array}{r} x-2 \\ x^2-x-6 \overline{) x^3-4x^2-7x+10} \\ -(x^3-x^2-6x) \\ \hline -2x^2+x+10 \end{array}$$

$\frac{-2x^2}{x^2}$

$y = x-2$ is slant asymptote

$\boxed{y = x-2 \text{ S.A.}}$



The numerator has zeros $x=-2,1,5$ so

$$\frac{x^3-4x^2-7x+10}{x^2-x-6}$$

$$R(x) = \frac{\cancel{(x+2)}(x-1)(x-5)}{(x-3)\cancel{(x+2)}} = \frac{(x-1)(x-5)}{x-3} = R^*(x)$$

& hole @ $x=-2$

$\mathcal{D}: \mathbb{R} \setminus \{-2, 3\}$

$\boxed{\text{V.A.}: x=3}$

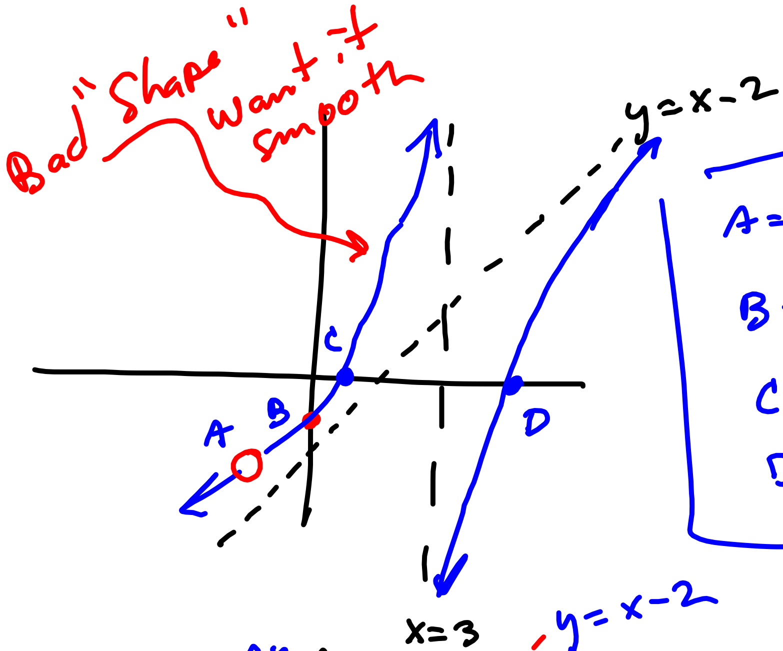
HOLE: $x=-2 \rightsquigarrow R^*(-2) = \frac{(-2-1)(-2-5)}{(-2-3)} = \frac{-3(-7)}{-5} = -\frac{21}{5}$

$\boxed{\left(-2, -\frac{21}{5}\right) = \text{HOLE}}$

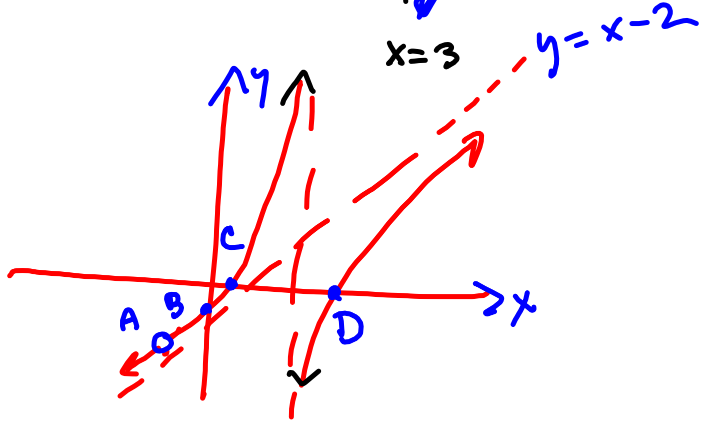
$\boxed{x\text{-ints}: (1,0), (5,0)}$

NOT $(-2,0)$, b/c HOLE!

y-int: $R^*(0) = \frac{10}{-6} = -\frac{5}{3} \leadsto \boxed{\left(0, -\frac{5}{3}\right) = \text{y-int}}$



$$\boxed{\begin{aligned} A &= \left(-2, -\tfrac{21}{5}\right) \\ B &= \left(0, -\tfrac{5}{3}\right) \\ C &= (1,0) \\ D &= (5,0) \end{aligned}}$$



§3.3 #75

$x^6 - x^5 - x^4 + x^3 - 12x^2 + 12x = 0$ Pull out common

$x(x^5 - x^4 - x^3 + x^2 - 12x + 12) = 0$

$a_5 = 1$ $a_0 = 12$

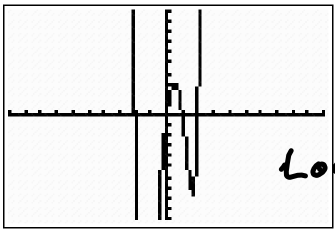
$\frac{p}{q}$: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$ possible rational zeros.

Descartes: $\underbrace{x^5 - x^4}_{1} \underbrace{- x^3 +}_{2} \underbrace{x^2 -}_{3} \underbrace{12x + 12}_{4} = f(x)$

4, 2 or 0 positive

$f(-x) = \underbrace{-x^5 - x^4}_{1} + x^3 + x^2 + 12x + 12$

1 negative zero



Bounds on real zeros: Meh.

Looks like $x = -2, 0, 1, 2$, from graph.

$x(x+2)(x^4 - 3x^3 + 5x^2 - 9x + 6)$

$$\begin{array}{r|rrrrrr} -2 & 1 & -1 & -1 & 1 & -12 & 12 \\ & & -2 & 6 & -10 & 18 & -12 \\ \hline & 1 & -3 & 5 & -9 & 6 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -2 & 1 & -3 & 5 & -9 & 6 \\ & & -2 & 10 & -30 & \text{No way!} \\ \hline & 1 & -5 & 15 & -39 & \end{array}$$

$x(x+2)(x-1)(x-2)(x^2+3)$

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & 5 & -9 & 6 \\ & & 1 & -2 & 3 & -6 \\ \hline 2 & 1 & -2 & 3 & -6 & 0 \\ & & 2 & 0 & 6 & \\ \hline & 1 & 0 & 3 & 0 & \end{array}$$

$x^2 + 3 = 0$

(No real solns)

$x^2 = -3$

$x = \pm i\sqrt{3}$

$x^2 + 3 = 0$

$a = 1, b = 0, c = 3$

$b^2 - 4ac = 0^2 - 4(1)(3)$

$= -12$

$x = \frac{\pm\sqrt{-12}}{2} = \frac{\pm i 2\sqrt{3}}{2}$

$= \pm i\sqrt{3}$

$x(x+2)(x-1)(x-2)(x - i\sqrt{3})(x + i\sqrt{3})$

$$\frac{12 \pm \sqrt{-32}}{8} =$$

$$\begin{array}{r|l} 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

$$= \frac{12 \pm 4i\sqrt{2}}{8} = \frac{4(3 \pm i\sqrt{2})}{8} = \frac{3 \pm i\sqrt{2}}{2}$$