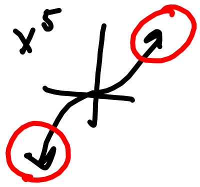
$$f(x) = x^5 - x^4 - 5x^3 + 9x^2 - 16x - 60$$

End behavior

x^5



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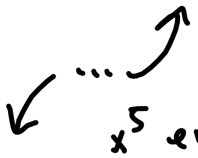


x^5 eventually dominates.

Descartes sez

$$\underbrace{x^5 -}_{1} x^4 \underbrace{- 5x^3 +}_{2} \underbrace{9x^2 -}_{3} 16x - 60$$

Top # $-2, -4, \ldots$

$\boxed{\text{3 or 1 positive zeros.}}$

(2) Negative zeros:

$$f(-x) = -x^5 - \underbrace{x^4 +}_{1} 5x^3 + 9x^2 + 16\underbrace{x -}_{2} 60$$

$$(-x)^5 = (-1x)^5 = (-1)^5 (x)^5 = -x^5$$

2 or 0 negative zeros

Ex 5 sign changes: 5, 3, or 1

8 " " : 8, 6, 4, 2, or 0

③ Rational zeros

$(2x+3)(5x-7)=0 \Rightarrow x=-\frac{3}{2}, \frac{7}{5}$

$= 10x^2+x-21$

$a_n = a_2 = 10$

$a_0 = -21$

$\frac{p}{q}$: $\frac{\text{factors of } 21}{\text{factors of } 10}$

So, guessing on $10x^2+x-21$:

$x = -\frac{3}{2}$: 3 is a factor of 21, 2 is a factor of 10

$10x^2 + x - 21$

$\frac{p}{q}$: $\boxed{\pm 1, \pm 3, \pm 7, \pm 21,}$
$\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{7}{2}, \pm\frac{21}{2},$
$\pm\frac{1}{5}, \pm\frac{3}{5}, \pm\frac{7}{5}, \pm\frac{21}{5}$
$\pm\frac{1}{10}, \pm\frac{3}{10}, \pm\frac{7}{10}, \pm\frac{21}{10}$

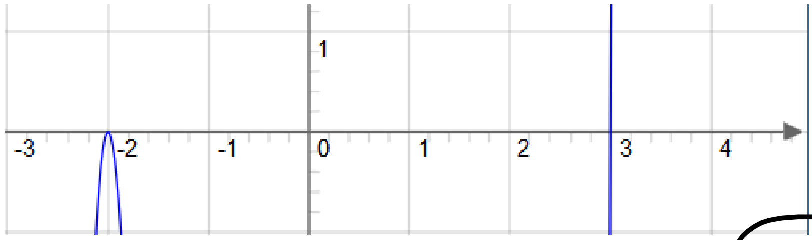
On TEST (sit-down), The nice ones tend to work.

$1x^5 - x^4 - 5x^3 + 9x^2 - 4x - 60$

p : 60
q : 1

$\frac{p}{q}$'s : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 15, \pm 20, \pm 30, \pm 60.$

$x=-2$, $x=3$ look good, from graph:



Split off $x+2$ - factor

$x = -2$

$(x+2)(x^4 - 3x^3 + x^2 + 7x - 30)$

$$\begin{array}{r|rrrrrr} -2 & 1 & -1 & -5 & 9 & -16 & -60 \\ & & -2 & 6 & -2 & -14 & +60 \\ \hline -2 & 1 & -3 & 1 & 7 & -30 & 0 \\ & & -2 & 10 & -22 & 30 & \\ \hline 3 & 1 & -5 & 11 & -15 & 0 & \\ & & 3 & -6 & 15 & & \\ \hline & 1 & -2 & 5 & 0 & & \end{array}$$

sweet

sweet!

$(x+2)^2(x^3 - 5x^2 + 11x - 15)$

sweet!

Factored over the reals

$\boxed{(x+2)^2(x-3)(x^2-2x+5)}$

irreducible quadratic factor.

$a=1,\ b=-2,\ c=5$

$b^2-4ac = (-2)^2 - 4(1)(5) = 4 - 20 = -16 < 0$

No more real zeros for $f(x)$.

So x^2-2x+5 is <u>irreducible</u> over the reals.

$$x = \frac{2 \pm \sqrt{-16}}{2(1)} = \frac{2 \pm i\sqrt{16}}{2} = \frac{2 \pm 4i}{2} = \frac{2(1 \pm 2i)}{2}$$

$$= \frac{1 \pm 2i}{1} = 1 \pm 2i$$

Factored over complexes!

$(x+2)^2(x-3)(x-(1+2i))(x-(1-2i))$

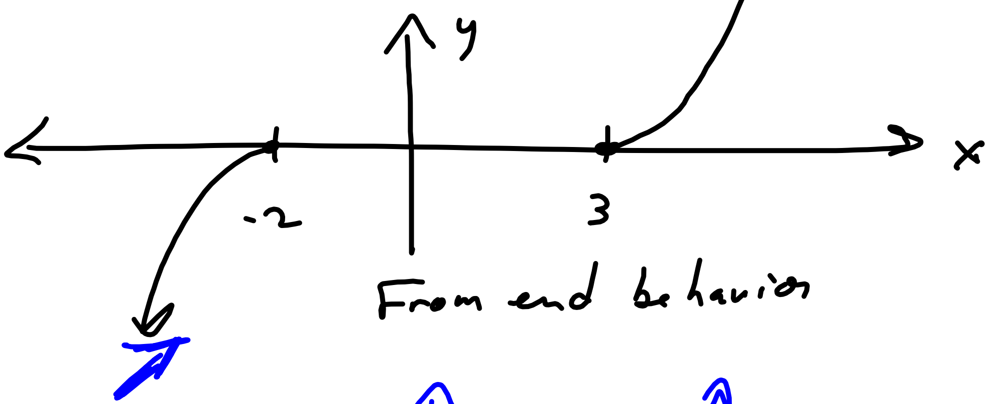
Split into linear factors!

$x = -2$, multiplicity 2
$x = 3$, " " 1
$x = 1+2i$ " " 1
$x = 1-2i$ " " 1

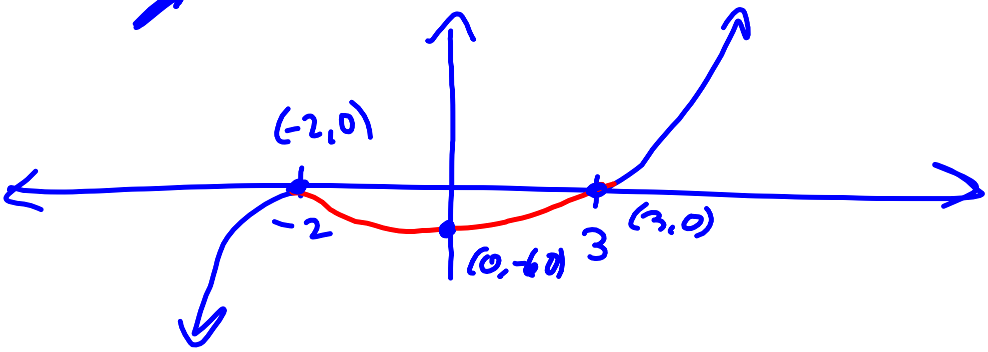
$\left.\right\}$ Adds up to 5

$x^5 + \cdots - 60$

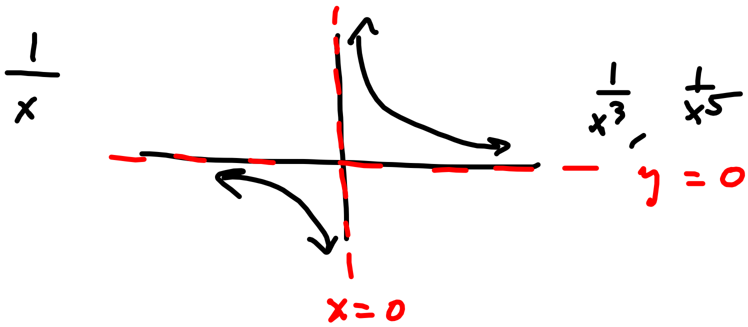
$(x+2)^2(x-3)(x^2-2x+5)$

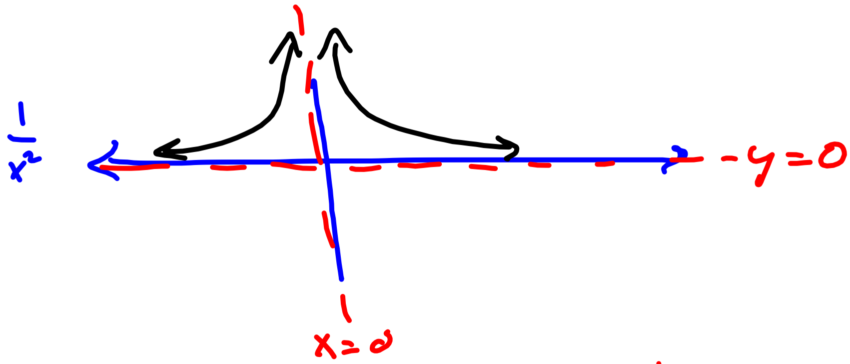


From end behavior



Analyzing the sign pattern:





Both are PROPER : $\dfrac{\text{Lower degree}}{\text{Higher degree}}$

$y = 0$ is horizontal asymptote for proper.

Impropers $\dfrac{\text{same degree}}{\text{same degree}}$

Horizontal Asymptote

$$y = \frac{\text{Big stuff}}{\text{Big stuff}}$$

$$\frac{3x^2+5x-6}{2x^2-17x+1000\pi} \xrightarrow{|x| \to \infty} \frac{3x^2}{2x^2} = \frac{3}{2} = y$$

$\dfrac{\text{Higher Degree}}{\text{Lower Degree}}$ Oblique/Slant Asymptote

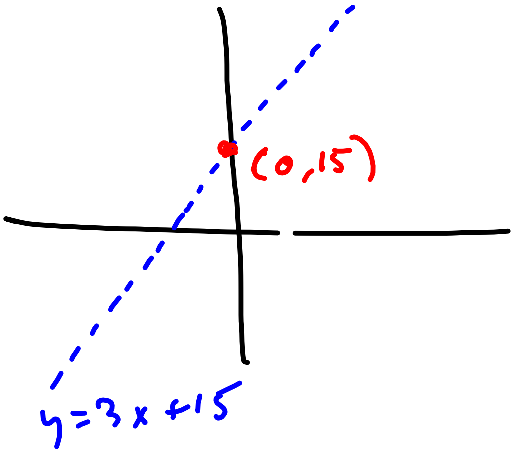
$$R(x) = \frac{3x^3+2x-1}{x^2-5x+2}$$

$$
\begin{array}{r|l}
 & 3x + 15 \quad r\ 71x - 31 \\
\hline
x^2-5x+2 & 3x^3 + 0x^2 + 2x - 1 \\
 & -(3x^3 - 15x^2 + 6x) \\
\hline
 & 15x^2 - 4x - 1 \\
 & -(15x^2 - 75x + 30) \\
\hline
 & 71x - 31
\end{array}
$$

So, $R(x) = \dfrac{3x^3+2x-1}{x^2-5x+1} = \boxed{3x+15} + \dfrac{71x-31}{x^2-5x+2}$

So, its end behavior is like $3x+15$

$$\frac{28}{3} = 9 + \frac{1}{3}$$



R(x) approaches $y = 3x+15$ off to left & off to right.

$$R(x) = \frac{2x^2-x-3}{x^2+2x-15} = \frac{(2x-3)(x+1)}{(x+5)(x-3)}$$

DOMAIN: $x \neq -5, 3$

No cancels, so

$\boxed{x=-5,\ x=3}$ Vertical Asymptotes

V.A.

Zeros: $x = -1, \frac{3}{2}$

x-ints: $(-1, 0)$, $(\frac{3}{2}, 0)$

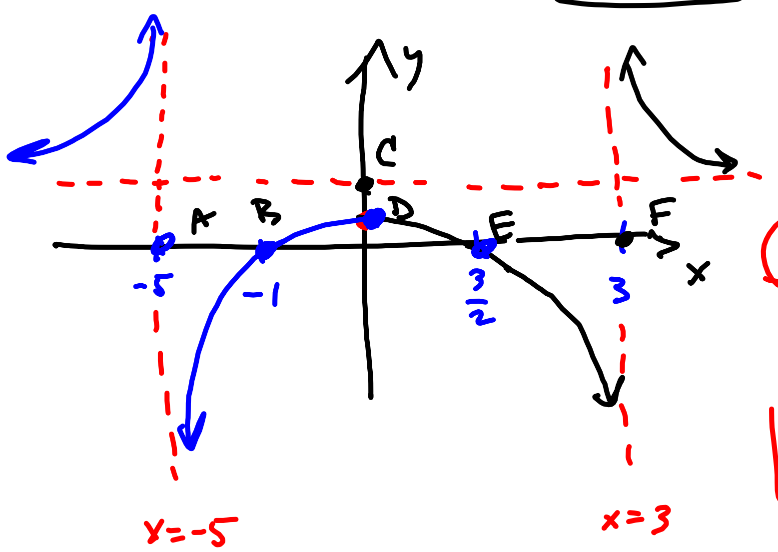
-5 $\quad -1$ $\quad \frac{3}{2}$ $\quad 3$

$=0$ $\quad =0$

H.A.: $x \to \pm\infty$

Top deg = Bottom Deg, so...

$\frac{2x^2}{x^2} = \boxed{2 = y}$ is Horizontal Asymptote,



y = 2

y-int:

$y = \frac{-3}{-15} = \frac{1}{5} = y$

$\boxed{\text{y-int}\ (0, \tfrac{1}{5})}$

$A = (-5, 0)$ $\quad E = (\frac{3}{2}, 0)$

$B = (-1, 0)$ $\quad F = (3, 0)$

$C = (0, 2)$

$D = (0, \frac{1}{5})$

$\frac{1}{x-3}$ changes sign @ $x = 3$