§3.3 #25

f has $x=-3, 5$ as zeros.

Find factored form of f.

$f(x) = (x-(-3))(x-5)$

$c = 3+2i \Rightarrow \bar{c} = \text{conjugate} = 3-2i$.

NOTE: $(3+2i) + (3-2i) = 6$ is real

Also (Key) $(3+2i)(3-2i) =$

$(a+b)(a-b) = a^2-b^2$

$(a+bi)(a-bi) = a^2-(bi)^2$

$= a^2-b^2i^2$

$= a^2+b^2$

Any time coefficients are real but one of the zeros is not, then the conjugate of the zero is also a zero.

$f(x) = 1x^4 - 5x^3 + x^2 + 49x - 78$ has zeros at $x = -3, 2, 3+2i$ & has real coefficients, so it factors this way:

$$(x+3)(x-2)(x-(3+2i))(x-(3-2i))$$

Write factored form of a poly with minimal degree AND REAL COEFFICIENTS & zeros: $1, -6, 2+3i$

CPT

$$(x-1)(x+6)(x-(2+3i))(x-(2-3i))$$

Find factored form of poly of degree 3, with real coefficients and zeros $x=1$, $x=-17$, $x=2-\sqrt{3}\,i$, if possible.

UNPOSSIBLE!

1 2 3 4

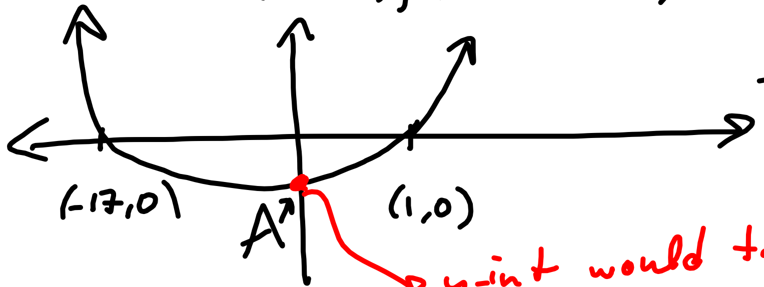
$(x-1)(x+17)(x-(2-\sqrt{3}\,i))\,(x-(2+\sqrt{3}\,i))$

$= x^4 + \text{smaller stuff}$

Needed but makes it degree 4, by Conjugate Pairs Theorem.

Graph

$(x-1)\ (x+17)\ \overbrace{(x-(2-\sqrt{3}\,i))\ (x-(2+\sqrt{3}\,i))}^{\text{work. Bleah}}$



y-int would take some work

You'd have to expand the $(x-(2-\sqrt{3}\,i))(x-(2+\sqrt{3}\,i))$

$= (x-2+\sqrt{3}\,i)(x-2-\sqrt{3}\,i)$

$= x^2 - 2x - \sqrt{3}\,i\,x - 2x + 4 + 2\sqrt{3}\,i + \sqrt{3}\,i\,x - 2\sqrt{3}\,i - \underbrace{(\sqrt{3}\,i)(\sqrt{3}\,i)}_{-3i^2}$

$= x^2 - 4x + 4 - 3i^2$

$= x^2 - 4x + 4 + 3 = x^2 - 4x + 7.$ Ugh!.

Now I can do y-int:

$f(0) = (0-1)(0+17)(0^2 - 4(0) - 7)$

$= (-1)(17)(-7) = 119 \rightarrow \boxed{A = (0, 119)}$

x^3+5x^2+7x+1

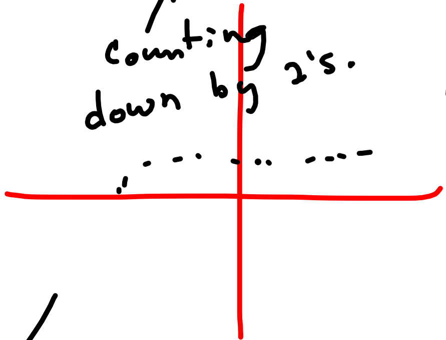
Descartes says: No positive zeros

The total # of sign changes in $f(-x)$

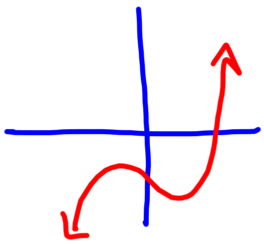
Negative zeros: $f(-x)$ 3 or 1 negative zeros

$= \underbrace{-x^3+5x^2}_{1}\underbrace{\ }_{2}\underbrace{-7x+1}_{3}$

counting down by 2's.

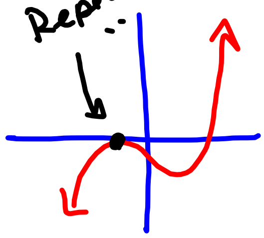


Cubic poly



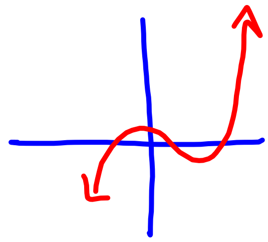
1 Real zero

Repeated Zero Root



SEEMS like 2 real zeros, but 3!

One is repeated



3 real, distinct zeros.