$f: \{(1,3),(2,7),(1,5),(3,-2)\}$

(a) Not func. B/c $(1,3)$ & $(1,5)$ pair $x=1$ w/ 2 values. ( f is not well-defined.)

(b) $\mathcal{D} = \{1,2,3,1\} = \{1,2,3\}$ redundant, but OK.

(c) $\mathcal{R} = \{3,7,5,-2\}$

(d) No. f isn't a func $\Longrightarrow$ Not a 1-to-1 func.

(2) $f(x) = \sqrt{x+4}$, $g(x) = \dfrac{7x^2 - 17x + \pi}{x+5}$

(a) $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x+4}}{\dfrac{7x^2 - 17x + \pi}{x+5}}$

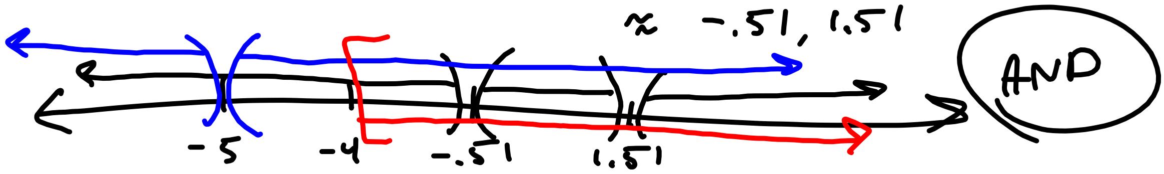
(b) $\boxed{\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \text{ and } g(x) \neq 0\}}$

$\mathcal{D}(f)$:
Need:
$x+4 \geq 0$
$\{x \mid x \geq -4\}$

$\mathcal{D}(g)$:
Need
$x+5 \neq 0$
$\{x \mid x \neq -5\}$

$g(x) \neq 0$
$7x^2 - 17x + \pi \neq 0$
$\Longrightarrow$
$x \neq \dfrac{17 \pm \sqrt{289 - 28\pi}}{14}$

$\approx -.51, 1.51$




$\approx [-4, -.51) \cup (-.51, 1.51) \cup (1.51, \infty)$

c) $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)+4}$

$f(x) = \sqrt{x+4}$

$$= \sqrt{\frac{7x^2 - 17x + \pi}{x+5} + 4}$$

$f(☺) = \sqrt{☺ + 4}$

$f(\square) = \sqrt{\square + 4}$

d) $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \neq -5 \text{ and } g(x) \geq -4$

$g(x) \geq -4$

$$\frac{7x^2 - 17x + \pi}{x+5} \geq -4$$

$$\frac{7x^2 - 17x + \pi}{x+5} + \left(\frac{4}{1}\right)\left(\frac{x+5}{x+5}\right) \geq 0$$

$$\frac{7x^2 - 17x + \pi + 4x + 20}{x+5} \geq 0$$

$$\frac{7x^2 - 13x + 20 + \pi}{x+5} \geq 0$$

$a = 7,\ b = -13,\ c = 20 + \pi$

$b^2 - 4ac = (-13)^2 - 4(7)(20 + \pi)$

$= 169 - 28\pi - 560 < 0$

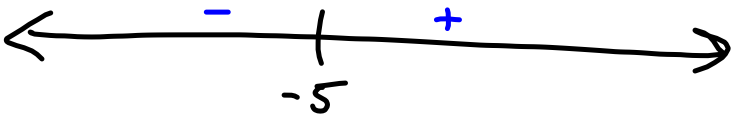
No Real zeros,

Never changes sign!

So

$\dfrac{7x^2 - 17x + \pi}{x+5}$ can ONLY

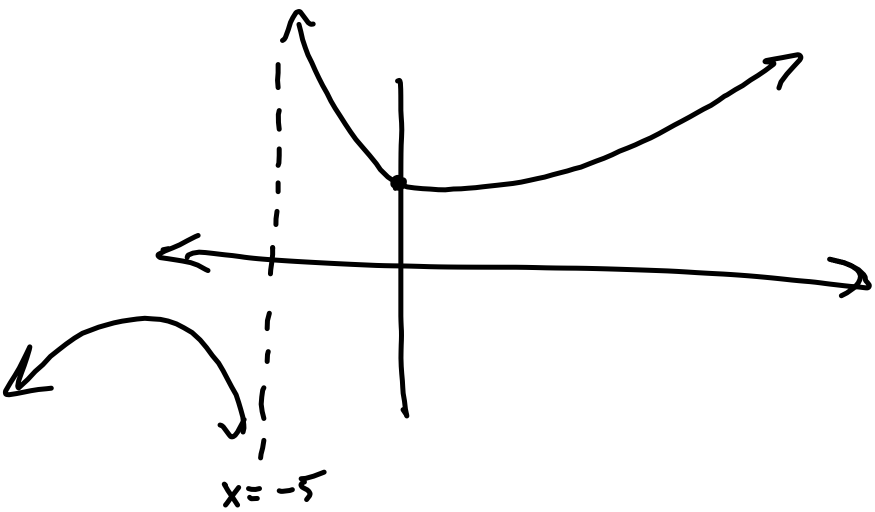
Change sign when $x+5$ does!




So, $x > -5$ does it!

$\{x \mid x > -5\} = \mathcal{D}(f \circ g)$.

$= (-5, \infty)$



Standard test question

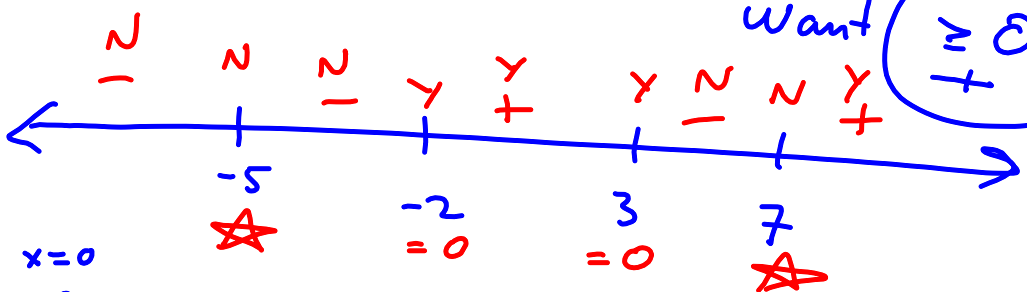
$$f(x) = \sqrt{\frac{(x+2)(x-3)}{(x-7)(x+5)^2}}$$

Find $\mathcal{D}(f)$

Need: $(x-7)(x+5)^2 \neq 0$ $\quad (x \neq 7, -5)$

$$\frac{(x+2)(x-3)}{(x-7)(x+5)^2} \geq 0$$

Want $\geq 0$ $+$




$x = 0$

OR

$x = 8$

OR

$x = -6$

$$\boxed{[-2, 3] \cup (7, \infty) = \mathcal{D}}$$

(4) $f(x) = 2x^2 - 3x$

$$\boxed{(x+h)^2 = x^2 + 2xh + h^2}$$

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

So $\dfrac{f(x+h) - f(x)}{h} =$

$h \to 0$, eventually

$$f(\boxed{x+h}) = 2\boxed{x+h}^2 - 3\boxed{x+h}$$

$$\frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$$

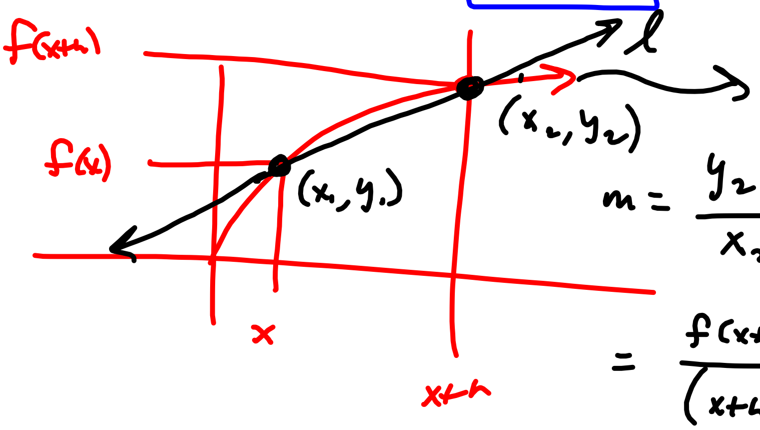
$$= \frac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 3h - 2x^2}{h}$$

$$= \frac{4xh + 2h^2 - 3h}{h} = \frac{h(4x + 2h - 3)}{h}$$

$$= \boxed{4x + 2h - 3}$$

Bonus: $h \to 0$ $\boxed{4x - 3}$




$f(x) = \sqrt{x}$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(x+h) - f(x)}{(x+h) - x}$$

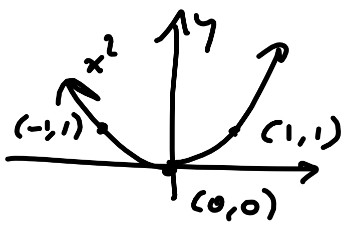
$$= \frac{f(x+h) - f(x)}{h}$$ is

the slope between 2 points on the graph of $f(x)$

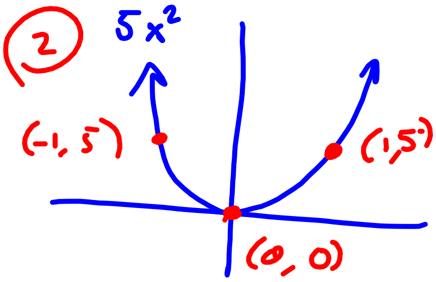
4 moves : $f(x) = x^2$

$5(x-2)^2 - 9$

| | |
|---|---|
| $\boxed{3x^2 \quad 3\ y\text{'s}}$ | |
| $(3x)^2$ | $\frac{1}{3}\ x\text{'s}$ |
| $(x-2)^2$ | $x+2$ |
| $x^2 - 5$ | $y-5$ |

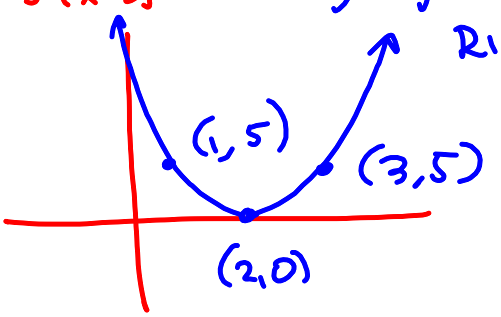


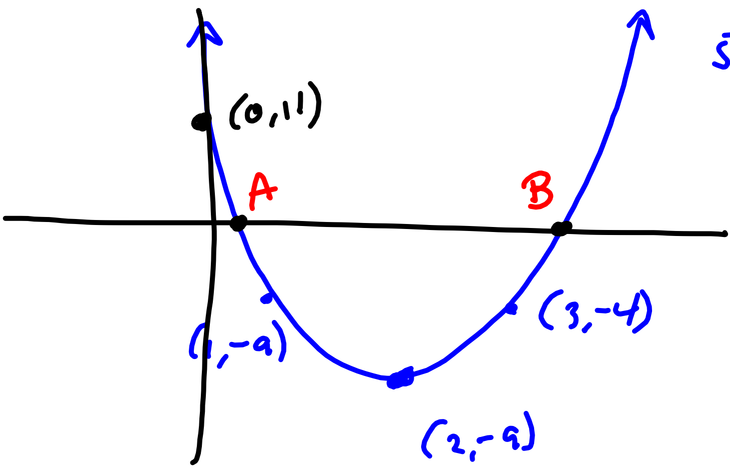

2 $5x^2$




$5(x-2)^2$ Delay by 2

RIGHT 2






$5(x-2)^2-9$

y-int:

$5(0-2)^2-9$

$= 5(-2)^2-9$

$= 20-9$

$= 11$

x-int:

$$5(x-2)^2-9=0$$

$$5(x-2)^2=9$$

$$(x-2)^2=\frac{9}{5}$$

$$x-2=\pm\sqrt{\frac{9}{5}}=\pm\frac{3}{\sqrt{5}}$$

$$=\pm\frac{3\sqrt{5}}{\sqrt{5}\sqrt{5}}=\pm\frac{3\sqrt{5}}{5}$$

$$x=2\pm\frac{3\sqrt{5}}{5}$$

$$\boxed{\begin{aligned} A &= \left(2-\frac{3\sqrt{5}}{5},0\right) \\ B &= \left(2+\frac{3\sqrt{5}}{5},0\right)\end{aligned}}$$

C3 stuff

$s^4-3s^2+6 \div s^2-5$

$\frac{s^4}{s^2}=s^2$

$$
\begin{array}{r}
s^2 \quad +2 \quad r\ 16 \\
s^2-5 \,\big)\, s^4+0s^3-3s^2+0x+6 \\
-\left(s^4 \qquad\quad -5s^2\right) \\
\hline
+2s^2+0x+6 \\
-\left(2s^2 \qquad -10\right) \\
\hline
16
\end{array}
$$

This says $s^4-3s^2+6$

$$= (s^2-5)(s^2+2)+16$$