Polynomials

$x = 2, -3, 5, 2-\sqrt{2}, 2+\sqrt{2}$ are zeros of $P(x)$. Give me its factored form:

$P(x) = 5(x-2)(x+3)(x-5)(x-(2-\sqrt{2}))(x-(2+\sqrt{2}))$

Synthetic Division:

Dividing Polynomial by x-c:

Divide x^2+5x+6 by $x+2$

$$\begin{array}{r|rrr} -2 & 1 & 5 & 6 \\ & & -2 & -6 \\ \hline & 1 & 3 & 0 \end{array}$$ Sweet!

x^1 c r

This says: $x^2+5x+6 = (x+2)(x+3)$

x^3-64. Divide by

$x^3-y^3 = (x-y)(x^2+xy+y^2)$

$x^3+y^3 = (x+y)(x^2-xy+y^2)$

Difference of 2 cubes

$x^3-64=0$

$x^3=64$

$\sqrt[3]{x^3} = x = \sqrt[3]{64} = 4$

So $x=4$ makes it zero!

$$\begin{array}{r|rrr} 4 & 1 & 0 & 0 & -64 \\ & & 4 & 16 & 64 \\ \hline & 1 & 4 & 16 & 0 \end{array}$$ sweet

x^2 x c r

This says $x^3-64 = (x-4)(x^2+4x+16)$

So, we got to "split off" the "linear" factor $(x-4)$ from $P(x) = x^3-64$.

$(x-4)(x^2+4x+16)$ $a=1, b=4, c=16$

$b^2-4ac = 4^2-4(1)(16) = -48$

Now, how to "split $P(x)$ into linear factors."

Find the zeros of $x^2+4x+16$, separately!

Not a perfect square. Can't factor with intermediate algebra skills.

$b^2-4ac = -48$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{-48}}{2}$$

$$= \frac{-4 \pm 2\cdot 2\sqrt{3}\, i}{2}$$

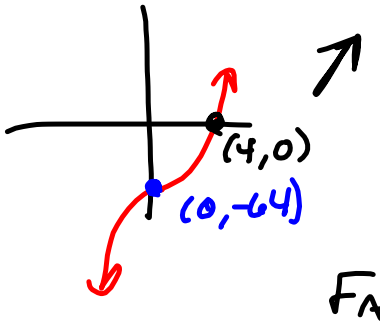
$$= \frac{4(-1 \pm i\sqrt{3})}{2}$$

$$= 2(-1 \pm i\sqrt{3})$$

$$= -2 \pm 2i\sqrt{3}$$

$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

$x^3 - 64$



Factor $P(x)$ all the way:

$(x-4)(x-(-2+2i\sqrt{3}))(x-(-2-2i\sqrt{3}))$

$i^2 = -1$

$\left(x - (-2+2i\sqrt{3})\right)\left(x - (-2-2i\sqrt{3})\right)$

$= (x+2-2i\sqrt{3})(x+2+2i\sqrt{3})$

$= x^2+2x+2i\sqrt{3}\,x+2x+4+4i\sqrt{3}$

$-2i\sqrt{3}\,x - 4i\sqrt{3} - 4i^2\sqrt{3}^2$

$= x^2+4x+4+4\cdot 3 = x^2+4x+16$ Sweet!

Test question:

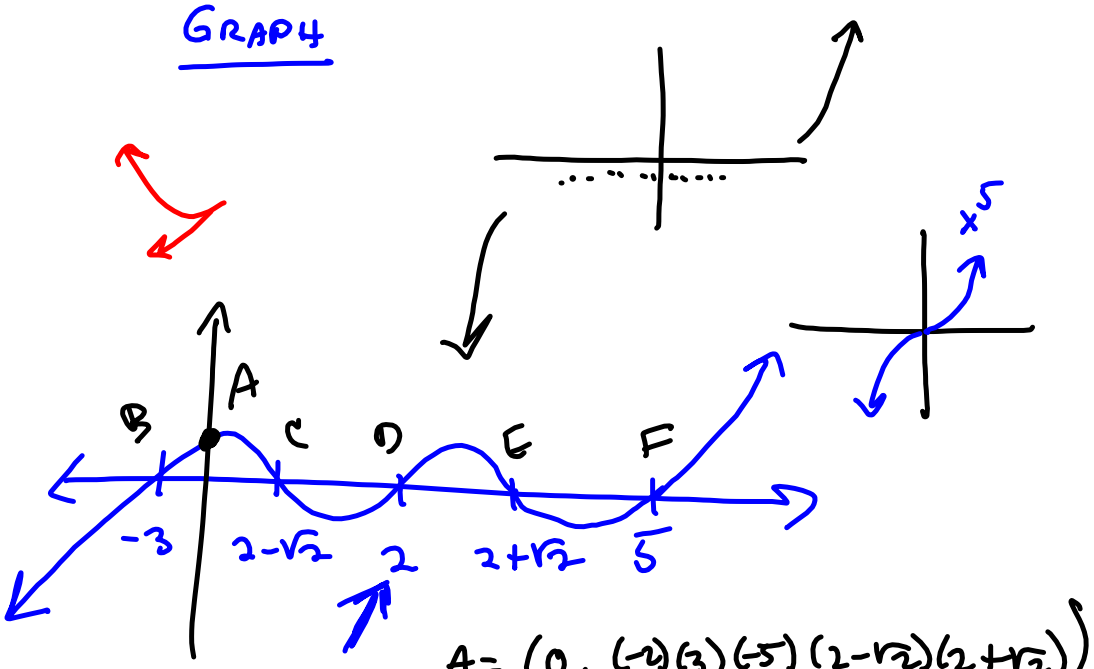
Expand $(x-(2+3i))(x-(2-3i))$

$-3, .6, 2, 3.14, 5$ $\quad 2-1.4=.6$ $\quad 2+1.4=3.14$

$P(x) = (x-2)(x+3)(x-5)^1(x-(2-\sqrt{2}))(x-(2+\sqrt{2}))^1$

$= x^5 + \text{other stuff}$

GRAPH



$A = (0, (-2)(3)(-5)(2-\sqrt{2})(2+\sqrt{2}))$

$B = (-3, 0)$ $\quad E = (2+\sqrt{2}, 0)$

$C = (2-\sqrt{2}, 0)$ $\quad F = (5, 0)$

$D = (2, 0)$