

Date, Time:

Do your own work on separate paper. Leave plenty of margin and plenty of room around your work. I'm not impressed if you squeeze more work into a smaller space. To the contrary. At the end, please make sure your problems are in order. I'm too old and ornery to want to go on a scavenger hunt to award you points.

1. (20 pts) Starting with $f(x) = 6^x$, sketch the graph of $g(x) = -2 \cdot 6^{-x^4} + 5$ in 5 steps (counting $f(x) = 6^x$ as the first step). Use $x = -1$, $x = 0$, and $x = 1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Finding the x - and y -intercepts is #2, so don't worry about them, until #2. Label each sketch as some variation on $f(x)$, for instance, $7 \cdot 2^{x^{11}} - 4$ would be $7f(x-11) - 4$.

2. (10 pts) Let $g(x) = -2 \cdot 6^{-x^4} + 5$. Find the x - and y -intercepts for this function, rounded to 4 decimal places. For 5 bonus points, label these intercepts on your final graph for #1.

3. Let $f(x) = \sqrt{x+2}$ and $g(x) = \frac{x+12}{x-3}$.

- a. (5 pts) What is the domain of f ?
 b. (5 pts) What is the domain of g ?
 c. (5 pts) Write the function $\frac{f}{g}$. Do not simplify.

d. (5 pts) What is the domain of $\frac{f}{g}$?

- e. (5 pts) Write the function $f \circ g$. Do not simplify.
 f. (5 pts) What is the domain of $f \circ g$?

4. Find the domain:

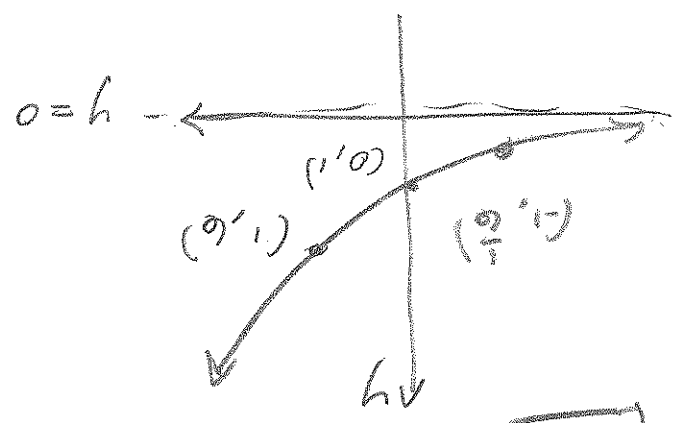
- a. (5 pts) $\sqrt{(x+3)^2(x-8)^3(x-12)}$. To speed up your sign pattern, it should be helpful to know that $(x+3)^2(x-8)^3(x-12) = -x^7 + 35x^6 - 423x^5 + 1625x^4 + 5132x^3 - 43680x^2 + 2304x + 276480$.
 b. (5 pts) $\log_3 \log_3 \left((x+3)^2(x-8)^3(x-12) \right)$ (Reinterpret previous sign pattern in the current context!)

5. Consider the equation $\log_3(x-3) + \log_3(x+3) = 3$.

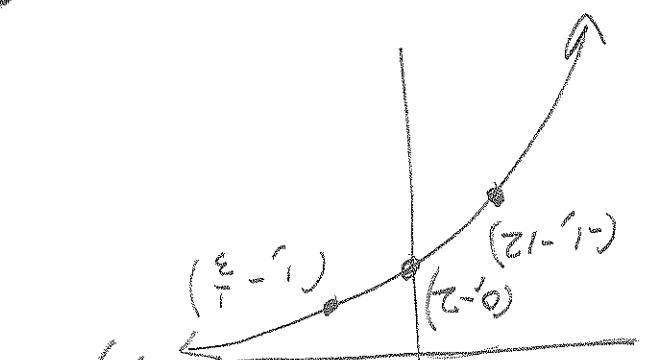
- a. (5 pts) What is the domain of this equation?
 b. (5 pts) Solve the equation.
 6. (10 pts) Solve $\pi^{-x-3} = e^{x+1}$. Give an exact answer and a decimal answer, rounded to 4 decimal places.

#1

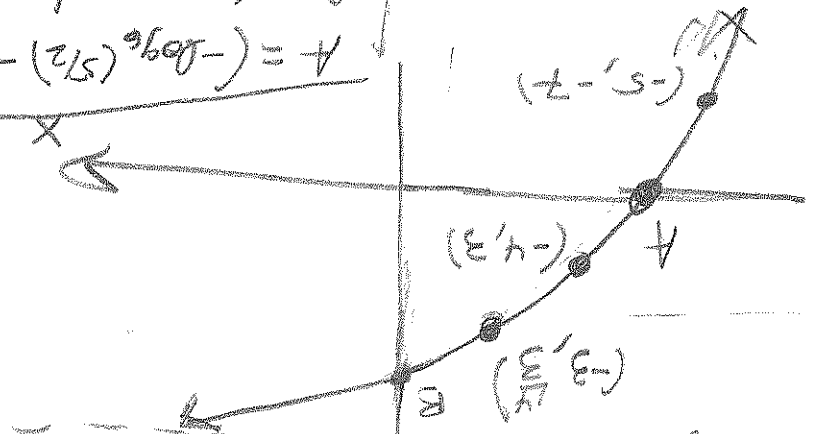
$f(x) = 6^x$



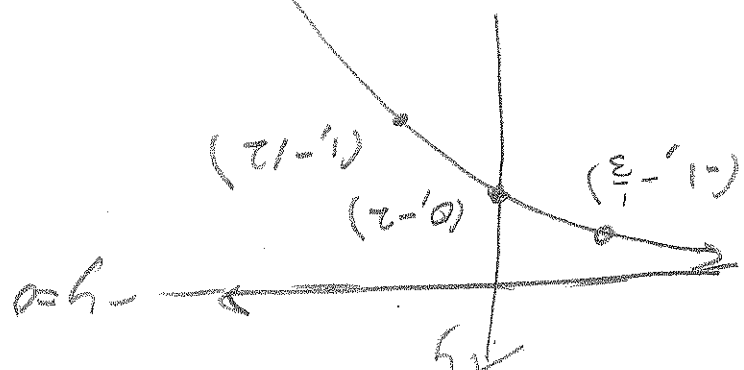
(3) $-x - 4 = -(x+4)$
 $-2f(-x) = -2 \cdot 6^{-x}$
 $y = 0$



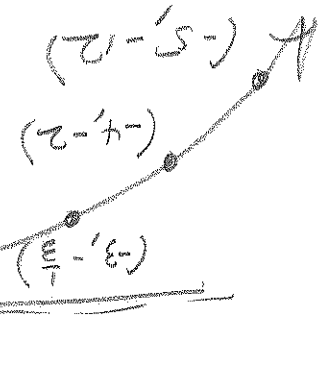
(5) $-2f(-(x+4)) + 5$
 $-2 \cdot 6^{-(x+4)} + 5 = g(x)$
 $-y = 5$



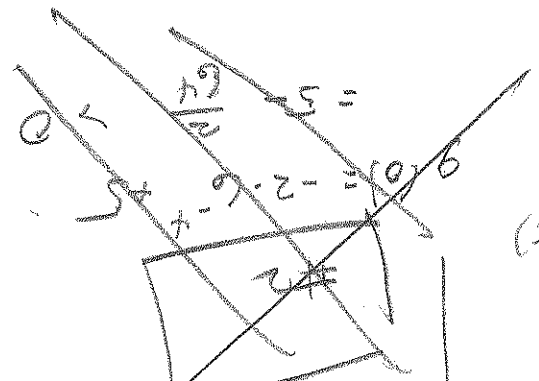
(2) $-2f(x) = -2 \cdot 6^x$



(4) $-2f(-(x+4)) = -2 \cdot 6^{-(x+4)}$



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 REFLECTION



Bonus $5 - \frac{3}{1} = \frac{5-1}{1} = \frac{4}{1} = 4$
 $5 - \frac{3}{1} = \frac{5-1}{1} = \frac{4}{1} = 4$

$A = (-\log_6(5/2) - 4, 0) \approx (-4.511391594, 0)$
 $B = (0, 5 - \frac{3}{1 - 3 \cdot 6^3}) = (0, \frac{3239}{648})$
 $X = (0, 4.99845679)$

$$\textcircled{a} \quad f(x) = \frac{1}{x} \mid x \in \mathbb{R} \setminus \{0\} \quad \text{or} \quad f(x) = \frac{1}{x}$$

$$\textcircled{b} \quad f(x) = \frac{x+2}{x-3}$$

$$\textcircled{c} \quad f(x) = \frac{1}{x} \mid x \in \mathbb{R} \setminus \{0\} \quad \text{or} \quad f(x) = \frac{1}{x}$$

$$\textcircled{d} \quad f(x) = \frac{1}{x} \mid x \in \mathbb{R} \setminus \{0\} \quad \text{or} \quad f(x) = \frac{1}{x}$$

$$\textcircled{e} \quad f(x) = \sqrt{x+2} \quad \text{or} \quad f(x) = \frac{x-3}{x+2}$$

TEST 4

$$= \frac{3239}{648}$$

$$= \frac{810}{1} + 5$$

$$= \frac{3 \cdot 6}{1} + 5$$

$$= \frac{6 \cdot 6}{2} + 5$$

$$= \frac{6}{2} + 5$$

$$= -2 \cdot 6 + 5$$

$$\textcircled{2} \quad g(x) = -2 \cdot 6 - 0 + 5$$

TEST 4

$$x = \log_{(5/2)}(5/2) - 4$$

$$-x = \log_{(5/2)}(5/2) + 4$$

$$-x - 4 = \log_{(5/2)}(5/2)$$

$$\frac{-x-4}{5} = \frac{6}{-x-4}$$

$$-2 \cdot 6 = -x - 4$$

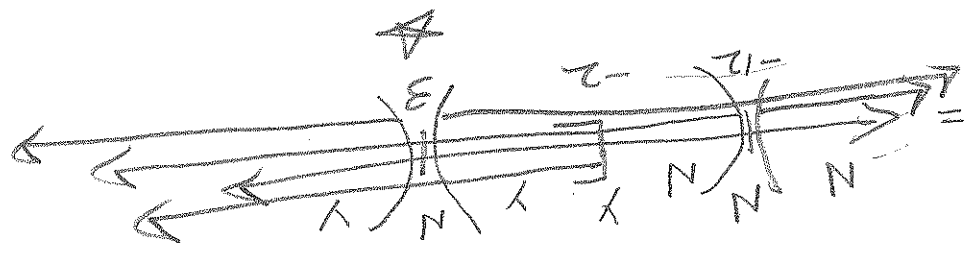
$$g(x) = 0$$

(3)

(3d) $\{x \in \mathbb{R} \mid x \in \mathbb{R} \setminus \{3\} \text{ and } g(x) \neq 0\}$

$= \{x \mid x \neq -2 \text{ and } x \neq -12 \text{ and } x \neq 3\}$

$(g(x) = 0 \Rightarrow \frac{x-3}{x+12} = 0 \Rightarrow x-3=0 \Rightarrow x=3)$



$= [-2, 3) \cup (3, \infty)$

(3c) $(f \circ g)(x) = f(g(x)) = \sqrt{\frac{x-3}{x+12} + 2}$

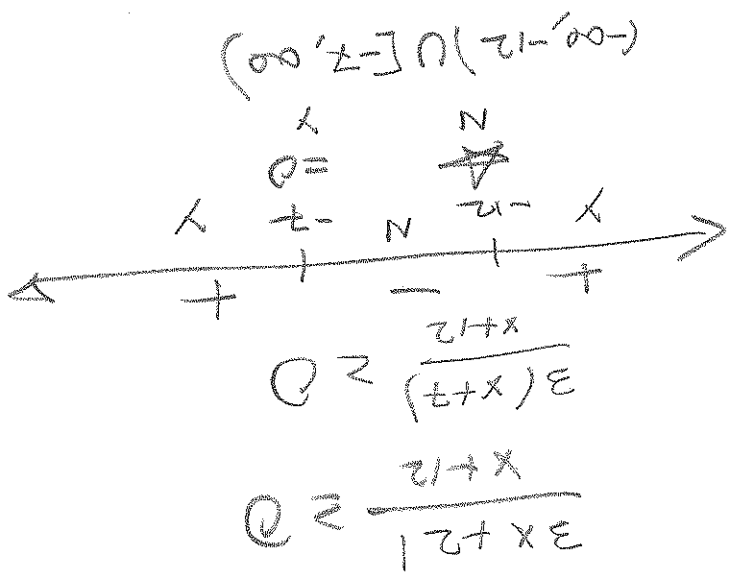
(3f) $\{x \in \mathbb{R} \mid x \in \mathbb{R} \setminus \{3\} \text{ and } g(x) \neq 0\}$

Sample 2

$\frac{x-3}{x+12} \geq -2$

$\frac{x-3}{2(x+12)} + \frac{x+12}{x+12} \geq 0$

$\frac{x-3+2x+24}{x+12} \geq 0$

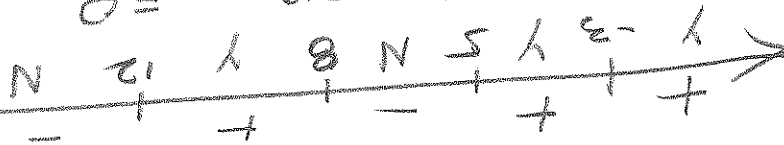


$(-\infty, -12) \cup [-2, \infty)$

$$(x+3)^2(x-8)(x-12)$$

$$(-\infty, -3) \cup (-3, 5) \cup (8, 12)$$

$$\begin{matrix} N & N & N & N \\ =0 & =0 & =0 & =0 \end{matrix}$$

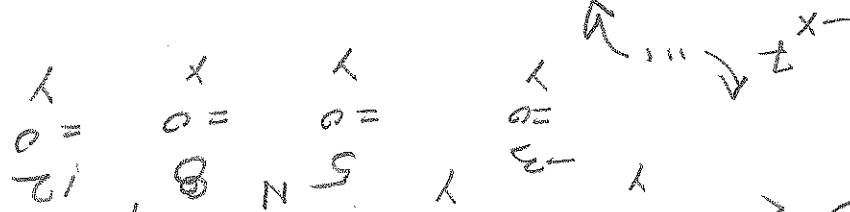


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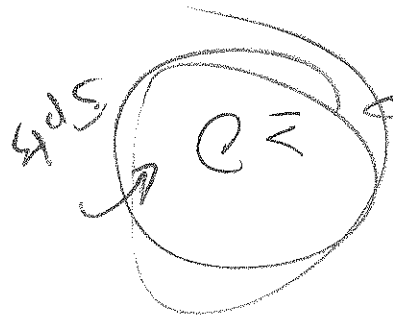


S.P.K

$$= (-\infty, 5] \cup [8, 12]$$



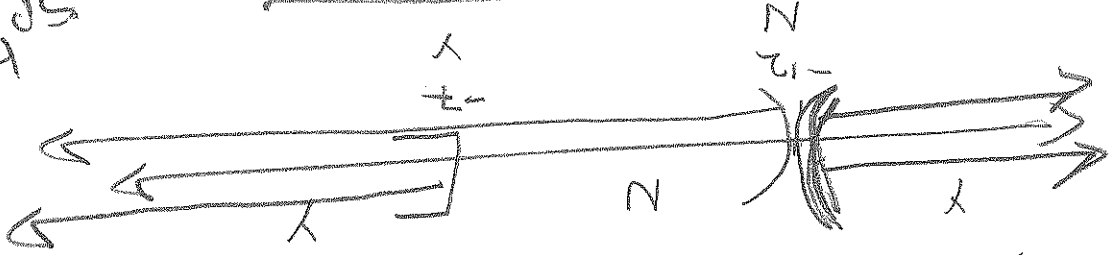
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S.P.K

$$= (-\infty, -12) \cup [7, \infty)$$

S.P.K



$$x \in (-\infty, -12) \cup [7, \infty)$$

and

$$x \in (-\infty, -12) \cup [7, \infty)$$

This means

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TEST 4

$$\{x \in \mathbb{R}\} \leftarrow \sigma \neq x$$

$$x = \sqrt{\epsilon} = \epsilon$$

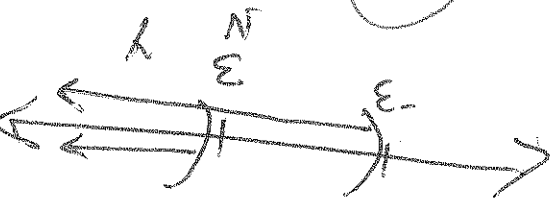
$$x = \epsilon$$

$$x = \epsilon$$

$$(x - \epsilon)(x + \epsilon) = \epsilon^2$$

$$\epsilon = \epsilon$$

$$\log_{\epsilon}((x - \epsilon)(x + \epsilon)) = \epsilon$$



$$(\epsilon, \infty)$$

$$\{x \mid x > \epsilon\}$$

$$\text{and } x + \epsilon > 0$$

$$\{x \mid x > \epsilon\}$$

$$\sigma = N \text{ and } x - \epsilon > 0$$

$$\log_{\epsilon}(x - \epsilon) + \log_{\epsilon}(x + \epsilon) = \epsilon$$

(6) $e^{\pi x - 3} = e^{\pi x + 1}$

$\ln(\pi x - 3) = \ln(e^{\pi x + 1})$

$(\ln(\pi))(x - 3) = \pi x + 1$

$\pi(x - 3) = \pi x + 1$

$2\pi x - 3\pi = \pi x + 1$

$\pi x - X = 3\pi + 1$

$X(\pi - 1) = 3\pi + 1$

$X = \frac{3\pi + 1}{\pi - 1}$

$\frac{30e^{\pi} + 1}{\ln(\pi) - 1} \approx 30.63769194$

(7)

$A_0 e^{3k} = 2A_0$

$e^{3k} = 2$

$3k = \ln(2)$
 $k = \frac{\ln(2)}{3}$

10pk

$k + = \ln\left(\frac{100}{3}\right)$
 $f = \frac{k}{\ln\left(\frac{100}{3}\right)}$

$e^{kt} = \frac{100000}{300} = \frac{100}{3}$

Given $A_0 = 300$
 want $300e^{kt} = 10,000$

15 yrs

$\frac{\ln\left(\frac{100}{3}\right)}{\ln\left(\frac{100}{3}\right)} \approx 15.17668107$

10pk

(B1)

$$3\pi x^{-3} = 5e^{x+2}$$

$$L(\text{LHS}) = L(\text{RHS})$$

$$L(3 \cdot \pi x^{-3}) = L(5e^{x+2})$$

$$L(3) + L(\pi x^{-3}) = L(5) + L(e^{x+2})$$

$$a + (L(\pi))(x^{-3}) = b + (L(e))(x+2) = b+x+2$$

$$a + a(x-3) = b+x+2$$

$$a + ax - 3a = b+x+2$$

$$ax - x = b+2-2+3a$$

$$x(a-1) = b+2-2+3a$$

$$= \frac{L(5)+2 - L(3) + 3L(\pi)}{L(\pi) - 1}$$

$$\approx 2.771917956$$

$$\approx 2.7719$$

B2

$$|7-3x| \geq 8$$

$$7-3x \geq 8 \quad \text{OR} \quad 7-3x \leq -8$$

$$-3x \geq 1 \quad \text{OR} \quad -3x \leq -15$$

$$\left\{ x \mid x \leq -\frac{5}{3} \right\} \quad \text{OR} \quad \left\{ x \mid x \geq 5 \right\}$$

$$= (-\infty, -\frac{5}{3}] \cup [5, \infty)$$

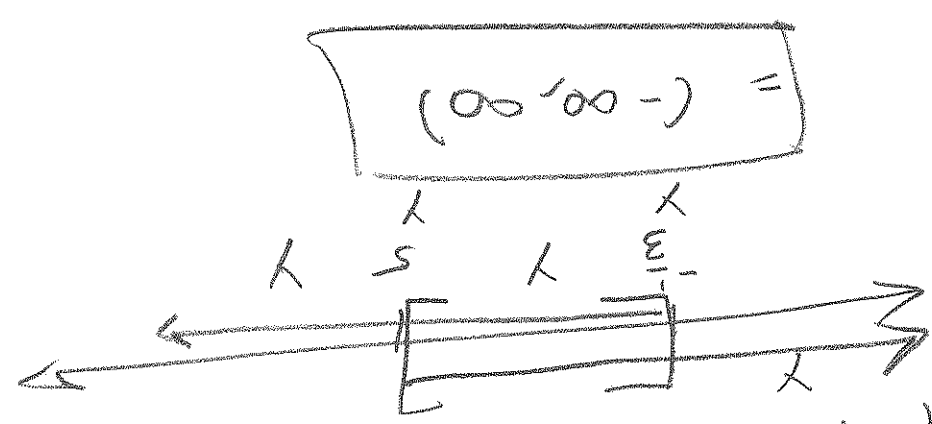
B3

$$|7-3x| \geq -8$$

$$7-3x \geq -8 \quad \text{OR} \quad 7-3x \leq +8$$

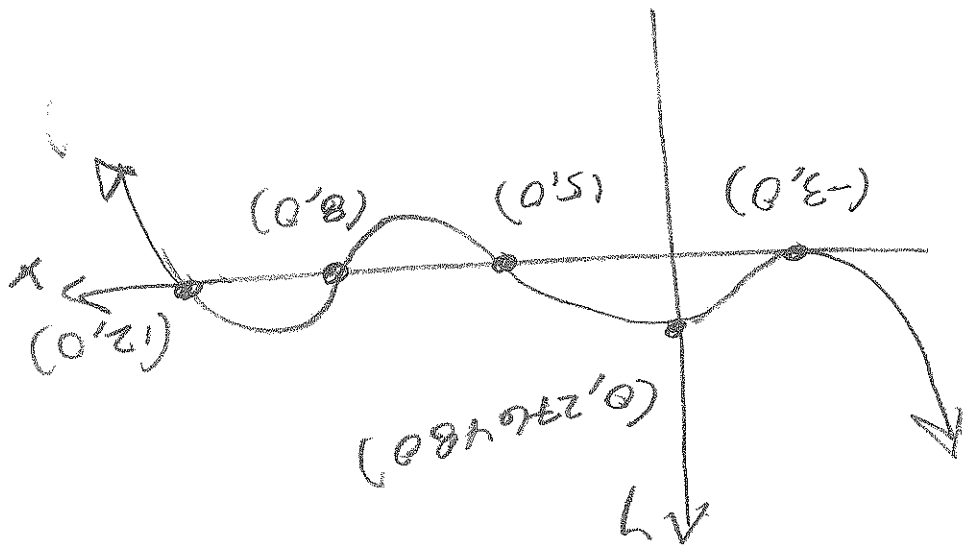
$$-3x \geq -15$$

$$\left\{ x \mid x \leq 5 \right\}$$



$$= (-\infty, \infty)$$

OR



$f(0) = 2.76480$
 $-x \neq \dots$



$$(x+3)^2 (5-x) (x-8)^3 (x-12)$$

B4

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