Recall

$S_n = a + ax + ax^2 + \dots + ax^{n-1} = a\left(\frac{1-x^n}{1-x}\right)$

Now, if $|x| < 1$, we can stretch this out to an infinite # of terms.

$2 + 2\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right)^2 = 2\left(\frac{1-\left(\frac{1}{2}\right)^3}{1-\frac{1}{2}}\right) = 2\left(\frac{1-\frac{1}{8}}{\frac{1}{2}}\right)$

$a = 2$

$x = \frac{1}{2}$

$n = ? = 3$ ($n-1$)

$= 2\left(\frac{\frac{7}{8}}{\frac{1}{2}}\right)$

$= 2\left(\frac{7}{8}\right)\left(\frac{2}{1}\right) = \frac{7}{2}$

$2 + 1 + \frac{1}{2} = 3 + \frac{1}{2} = \frac{6}{2} + \frac{1}{2} = \frac{7}{2}$ ✓

Annuity: Think of each payment as a separate savings account

n	n-1	.....	3	2	1

$R + R(1+i) + \dots + R(1+i)^{n-3} + R(1+i)^{n-2} + R(1+i)^{n-1}$

It's Geometric

$a = R$

$x = (1+i)$

$n = n$

So after n periods, you have

$a\left(\frac{1-x^n}{1-x}\right)$

$= R\left[\frac{1-(1+i)^n}{1-(1+i)}\right]$

$1-(1+i) = 1-1-i = -i$

$= R\left[\frac{(1+i)^n - 1}{i}\right]$ = F.V. of the annuity

$= R\left[\frac{(1+i)^n - 1}{i}\right] = FV$

Suppose this annuity is ACTUALLY you paying off a loan and your bank sells the loan to another bank. How do the bankers decide the price of that loan? What's it worth, today?

It's worth what the banker who bought it could earn by investing at the same rate in one lump sum.

$$P(1+i)^n = R\left[\frac{(1+i)^n - 1}{i}\right]$$

↑ lump sum P

Solve for P

Variations: What will the payments be if you borrowed P dollars, today?

Solve for R

PV = Present Value of the Loan = P

$$P = PV = \boxed{(1+i)^{-n}} R\left[\frac{(1+i)^n - 1}{i}\right]$$

Payment Calculator

$\frac{1}{(1+i)^n} = (1+i)^{-n}$

$$P(1+i)^n\left[\frac{i}{(1+i)^n - 1}\right] = R$$

$$PV = (1+i)^{-n} R\left[\frac{(1+i)^n - 1}{i}\right]$$

I'm gonna buy your annuity

OR . . . "How much did you borrow?"

$R = \$500$

$r = .07$

$m = 12$

$t = 5$

$n = mt = (12)(5)$

$i = \frac{r}{m} = \frac{.07}{12}$

```
(1+.07/12)^(-12*
5)*500*((1+.07/1
2)^(12*5)-1)/(.0
7/12)
      25250.99675
```

$$P(1+i)^n = R\left[\frac{(1+i)^n - 1}{i}\right]$$

I'll give you

$$A = P(1+i)^n \quad \& \quad FV = R\left[\frac{(1+i)^n - 1}{i}\right]$$

And you need to know that buying an annuity involves

$$A = FV$$

Get a calculator that allows you to see the entries & recapture them.

FINAL EXAM

COMPREHENSIVE

WED 7:10 - 9:00

↳ EARLY!