P = Principal or Present Value of Savings
r = annual rate of interest
m = # of periods per year
t = # of years
i = interest rate per period $= \frac{r}{m}$
n = total # of periods = mt
A = future value of savings account
S = " " " ANNUITY

Theory of compound interest : Savings Account

At the end of Period...	The Balance is...
1	$P + Pi = P(1+i)$
2	$P(1+i) + (P(1+i))i = P(1+i)[1+i] = P(1+i)^2$
3	$P(1+i)^2 + (P(1+i)^2)i = P(1+i)^2(1+i) = P(1+i)^3$
4	$P(1+i)^4$
	$\vdots$
n	$\boxed{A = P(1+i)^n = A} = P\left(1+\frac{r}{m}\right)^{mt}$

\$10,000 invested at 5% APR for 6 years, compounded monthly

$r = .05$ $P = 10000$

$m = 12$

$t = 6$

$$A = P(1+i)^n = P\left(1+\frac{r}{m}\right)^{mt}$$

$$= 10000\left(1+\frac{.05}{12}\right)^{(12)(6)}$$

$$\boxed{\approx \$13,490.18}$$

```
10000*(1+.05/12)
^(12*6)
          13490.17744
```

$$S_n = a + ax + ax^2 + ax^3 + \dots + ax^{n-1}$$
$$-\quad xS_n = \qquad ax + ax^2 + ax^3 + \dots + ax^{n-1} + ax^n$$

$$S_n - xS_n = a - ax^n$$

$$S_n(1-x) = a(1-x^n)$$

$$S_n = \frac{a(1-x^n)}{1-x}$$ = Geometric Series Formula.

$$\overset{1}{3} + \overset{2}{3\cdot\frac{1}{4}} + \overset{3}{3\cdot\frac{1}{16}} + \overset{4}{3\cdot\frac{1}{64}}$$

$a = 3$

$x = \frac{1}{4}$ = Common ratio.

$$S_4 = \frac{3\left(1-\left(\frac{1}{4}\right)^4\right)}{1-\frac{1}{4}} = \frac{3\left(1-\frac{1}{256}\right)}{\frac{3}{4}}$$

$$= \frac{3\left(\frac{256-1}{256}\right)}{\frac{3}{4}} = 3\left(\frac{255}{256}\right)\cdot\frac{4}{3}$$

$$= \frac{255}{256}\cdot 4 = \frac{255}{64}$$

```
^(12*6)
            13490.17744
3+3*1/4+3*(1/4)²
+3*(1/4)^3
               3.984375
Ans▸Frac
                 255/64
```

Money goes in @ end of month.
Interest is calculated at the end of the month on the Balance at the beginning of the month.

R = Monthly Payment

At the end of Period	This is the Balance
1	R
2	$R + Ri + R = R(1+i) + R$
3	$R(1+i)+R + (R(1+i)+R)i + R$
	$= (R(1+i)+R)(1+i) + R$
	$= R(1+i)^2 + R(1+i) + R$
$\vdots$	$\vdots$
n	$R + R(1+i) + R(1+i)^2 + \dots + R(1+i)^{n-1}$

2nd pmt

3rd pmt

Previous Balance

Interest on Previous Balance.

See? It's Geometric!

$$= R\left(\frac{1-(1+i)^n}{1-(1+i)}\right)$$

$a = R$

$x = (1+i)$

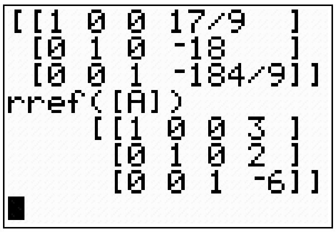
$$S_n = \frac{a(1-x^n)}{1-x}$$

Geometric Series!

$$\boxed{S_n = \frac{R((1+i)^n - 1)}{i} = \text{Future Value of an Annuity.}}$$

#17 from

$$5x - y + z = 7$$
$$3x + 2y - 3z = 31$$
$$x - 3y + 2z = 15$$



(1) $x - 3y + 2z = -15$
(2) $3x + 2y - 3z = 31$
(3) $5x - y + z = 7$

$$\begin{array}{lr} -3(1) & -3x + 9y - 6z = -45 \\ (2) & 3x + 2y - 3z = 31 \\ \hline & 11y - 9z = -14 \end{array}$$

$$\begin{array}{lr} -5(1) & -5x + 15y - 10z = -75 \\ (3) & 5x - y + z = 7 \\ \hline & 14y - 9z = -68 \end{array}$$

(1) $11y - 9z = -14$
(2) $14y - 9z = -68$

$$\begin{array}{lr} -(1) & -11y + 9z = 14 \\ (2) & 14y - 9z = -68 \\ \hline & 3y = -54 \end{array}$$

$$y = \frac{-54}{3} = \boxed{-18 = y}$$

Back-substitute:

$14y - 9z = -68$

$14(-18) - 9z = -68$

$-252 - 9z = -68$

$-9z = 184$

$$\begin{array}{r} 180 \\ 72 \\ \hline 252 \\ -68 \\ \hline 184 \end{array} \qquad \begin{array}{r} {}^{3}18 \\ 14 \\ \hline 72 \\ 180 \\ \hline 2 \end{array}$$