

From Practice Final, something

$$\sum_{n=1}^{\infty} 3 \cdot \left(\frac{1}{2}\right)^{n-1} = 3 + 3\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + \dots$$

$3\left(\frac{1}{2}\right)^0 = 3 \cdot 1 = 3$

$$\sum ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

$$S_n = \sum_{k=1}^n 3 \cdot \left(\frac{1}{2}\right)^{k-1} = 3 + 3\left(\frac{1}{2}\right) + \dots + 3\left(\frac{1}{2}\right)^{n-1}$$

$$= 3 \left(\frac{1 - \left(\frac{1}{2}\right)^n}{1 - \frac{1}{2}} \right)$$

$$\sum_{k=1}^{\infty} 3 \cdot \left(\frac{1}{2}\right)^{k-1} = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \sum_{k=1}^n 3 \cdot \left(\frac{1}{2}\right)^{k-1}$$

$$= \lim_{n \rightarrow \infty} 3 \left(\frac{1 - \left(\frac{1}{2}\right)^n}{1 - \frac{1}{2}} \right) = 3 \left(\frac{1}{1 - \frac{1}{2}} \right)$$



$\frac{1}{2} \left(\frac{1}{1-r} \right)$
for $\sum_{k=1}^{\infty} ar^{k-1}$

Annuities grow geometrically

$$S'_n = R + R(1+i) + R(1+i)^2 + \dots + R(1+i)^{n-1}$$

$$= \sum_{k=1}^n R(1+i)^{k-1} = R \left(\frac{1 - (1+i)^n}{1 - (1+i)} \right)$$

$$= FV = R \left(\frac{(1+i)^n - 1}{i} \right) \text{ on cheat sheet.}$$

Saving for kids' college
common ratio $(1+i) = x = 1+i$
reserve r for interest rate

$$A = FV$$

$$P(1+i)^n = R \left(\frac{(1+i)^n - 1}{i} \right)$$

Price of the annuity (Amount of Loan) Payments.

$$|A| < B$$

$$A < B \text{ AND } A > -B$$

$$|A| > B$$

$$A > B \text{ OR } A < -B$$

$(x^3)(x)(x)^2 = x^{3+1+2} = x^6 \rightarrow \dots \rightarrow$
 $(x-2)^3(x+1)(x-4)^2 > 0$

Sign chart for $(x-2)^3(x+1)(x-4)^2 > 0$:
 Critical points: $x = -1, 2, 4$
 Intervals: $(-\infty, -1)$ (+), $(-1, 2)$ (-), $(2, 4)$ (+), $(4, \infty)$ (+)
 Zeros: $x = -1, 2, 4$
 Multiplicities: $x = -1$ (No), $x = 2$ (No), $x = 4$ (No)

(x-4)² doesn't change sign
 (x-4)² doesn't change sign
 (x-2)³(x+1)(x-4)²

$= (-\infty, -1) \cup (2, 4) \cup (4, \infty)$

$\frac{(x+1)(x-4)^2}{(x-2)^3} \geq 0$

Sign chart for $\frac{(x+1)(x-4)^2}{(x-2)^3} \geq 0$:
 Critical points: $x = -1, 2, 4$
 Intervals: $(-\infty, -1)$ (+), $(-1, 2)$ (-), $(2, 4)$ (+), $(4, \infty)$ (+)
 Zeros: $x = -1, 4$
 Multiplicities: $x = -1$ (No), $x = 2$ (No), $x = 4$ (No)

$= (-\infty, -1] \cup (2, \infty)$