S1.2 Models

Formulas

Solve

Ax + Hy = J, for x.

$Ax + Hy = J$

$-Hy = -Hy$

$Ax = J - Hy$

optional

$\frac{Ax}{A} = \frac{J - Hy}{A}$

$x = \frac{J - Hy}{A}$

$Ax + Hy = J$

$Ax = J - Hy$

$x = \frac{J - Hy}{A}$

17. $S = \frac{a_1 - a_1 r^n}{1 - r}$ for a_1 (geometric series)

M1 LCD = 1 − r

$\frac{S(1-r)}{1-r} = \frac{a_1 - a_1 r^n}{1-r}$

$S(1-r) = a_1 - a_1 r^n$

$a_1 - a_1 r^n = S(1-r)$

$a_1(1 - r^n) = S(1-r)$

$\frac{a_1(1-r^n)}{1-r^n} = \frac{S(1-r)}{1-r^n}$

$a_1 = \frac{S(1-r)}{1-r^n}$

M2 Clear fractions

$S = \frac{a_1 - a_1 r^n}{1-r}$

$S(1-r) = \left(\frac{a - a_1 r^n}{1-r}\right)(1-r)$

$S(1-r) = a - a_1 r^n$

$\left(S = \frac{a_1 - a_1 r^n}{1-r}\right)(1-r)$

$S(1-r) = a_1 - a_1 r^n$

Mixture Problems – I do NOT do these like the book. I just go ahead and use 2 variables, rather than hide from you what's going on. My way is tougher, at first, perhaps, but you want a PROCESS that will carry you.

Two Equations : Amt of Pure = Amt of Pure
Total Amt = Total Amt

Two Variables x = amt of the one thing (give units)
y = " " " other thing (" ")

$ax + by$ = Total pure (a & b are concentrations or unit prices or . . .)

$x + y$ = TOTAL

Maybe a bridge to my method is charting it out

54. *Mixing Alcohol Solutions* A pharmacist needs to obtain a 70% alcohol solution. How many ounces of a 30% alcohol solution must be mixed with 40 ounces of an 80% alcohol solution to obtain a 70% alcohol solution?

HINT Add x ounces of 30% solution to 40 ounces of 80% solution to get $x + 40$ ounces of 70% solution.

x = Amt of 30% Alcohol (ounces)

Pure alco. = Pure Alco. $.3x + .8(40) = .7(x+40)$

Total Mix = Total Mix $x + 40 = x + 40$

If amt of 80% was ALSO unknown & Final Volume WAS known, then it's a 2-variable problem.

Suppose Final Volume were to be 100 ounces.

Then x = Amt of 30% alc. (oz)
y = " " 80% alc. (oz)

Pure Alc. $.3x + .8y = .7(100)$

Total Mix, $x + y = 100$

Unit Analysis: $.3x$ means $\left(\frac{.3 \text{ oz pure alc}}{(1 \text{ ~~oz of mix~~})}\right)\left(x \text{ (~~oz of mix~~)}\right)$

$= .3x$ oz of pure alcohol.

55. *Harvesting Wheat* With the old combine, Nikita's entire wheat crop can be harvested in 72 hr, but a new combine can do the same job in 48 hr. How many hours would it take to harvest the crop with both combines operating?

HINT The rate for the old combine is 1/72 crop/hr and for the new one it is 1/48 crop/hr. Together the rate is $1/x$ crop/hr.

Let x = time it takes for the two machines working together (in hours)

Your Book says write an equation for what gets done in one time unit (in this case, 1 hour).

If it takes 10 hrs to get the job done, then $\frac{1}{10}$ of the job is done in 1 hr.

In one hour, then,

$\frac{1}{72} + \frac{1}{48} = \frac{1}{x}$

$\frac{1}{72} + \frac{1}{48} = \frac{1}{x}$ one-hour's worth.

1 job done = 1 job done

My approach is 1 job done = 1 job done

$\frac{1}{72}x + \frac{1}{48}x = 1$ job

$\left(\frac{\frac{1}{72} \text{ of job done}}{1 \text{ hr}}\right)(x \text{ hrs})$

Let x = amt of time spent harvesting by the 1st harvester (in hours.

Since they're working together and both start & end at the same time, x = amt of time spent harvesting by the 2nd harvester.

$\frac{1}{72}x + \frac{1}{48}x = 1$

Check out #s 57 & 58

57. *Batman and Robin* Batman can clean up all of the crime in Gotham City in 8 hr working alone. Robin can do the same job alone in 12 hr. If Robin starts crime fighting at 8 A.M. and Batman joins him at 10 A.M., then at what time will they have all of the crime cleaned up?

58. *Scraping Barnacles* Della can scrape the barnacles from a 70-ft yacht in 10 hr using an electric barnacle scraper. Don can do the same job in 15 hr using a manual barnacle scraper. If Don starts scraping at noon and Della joins him at 3 P.M., then at what time will they finish the job?

Let x = the amt of time Don spends working (hrs)

y = " " " " Della " " = x-3 (hrs)

$\frac{1}{15}x + \frac{1}{10}(x-3) = 1$

find x, add x to Don's 12:00 start time to get the finish time.

What is the simple interest rate if $109.45 in interest is earned on a deposit of $1662.47 in one year?

The simple interest rate is ☐%.
(Round to the nearest tenth of a percent.)

$I = Prt$

$109.45 = 1662.47r \cdot (1)$

$\frac{109.45}{1662.47} \approx .065835774 48$

$= 6.5835774 48\%$

$\approx 6.6\%$

Cameron and his friend John bought a used circus carousel for $65,721, including sales tax. If the sales tax rate is 7%, then what was the cost of the carousel before the tax?

Price After Tax = Price B4 tax + tax

$65,721 = x + .07x = 1.07x$

$\frac{65721}{1.07} = x \approx 61421.49533$

$\approx \$61421.50$

$61421.5 + .07(61421.5) = 65721$

x = price before tax.

People try to do $(65721)(.07)$ & that's no help.

Most common error

$65721 - .07(65721)$

≈ 61120.53

$61120.53 + .07(61120.53)$

$= 65398.9671$

a paid one-half of her game-show winnings to the government for taxes. She invested one-third of her winnings in Jeff"s copy shop at 19% interest and one-sixth er winnings in Kaiser's German Bakery at 12% interest. If she earned a total of $5,250 on the investments in one year, then how much did she win on the game w?

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x = how much she won ($)

Lost $\frac{1}{2}$ to gummint

$\frac{1}{2}x$ to invest

$\frac{1}{3}(x)$ invested at 19%

$\frac{1}{6}(x)$ " " 12%

Total Interest = Total Interest

$(.19)(\frac{1}{3}x)+(.12)(\frac{1}{6}x)=5250$

$\left(\frac{.19x}{3}+\frac{.12x}{6}=5250\right)18$

$(6)(.19x)+(3)(.12x)=(5250)(18)$

etc.

Bobby and Rick are in a 16-lap race on a one-mile oval track. Bobby, averaging 92 mph, has completed six laps just as Rick is getting his car onto the track. What speed does Rick have to average to be even with Bobby at the end of the sixteenth lap?

16 mile race

Bobby averages 92 mph

Rick starts when Bobby is 6 laps into it.

	r	t	D
Bobby	92	t_1	16
Rick	x	t_2	16

Gotta cover 16 miles in same amount of time as Bobby