The plan is to have another couple (or few) videos, after

- This One - Prep for the Chapter 4 Test
- Next One - Review a previous Chapter 3 Test (Assign a re-take/take-home for Chapter 3)
- Review Test Material for the segment between Test 4 and the Final Test
- Go over an old Final.

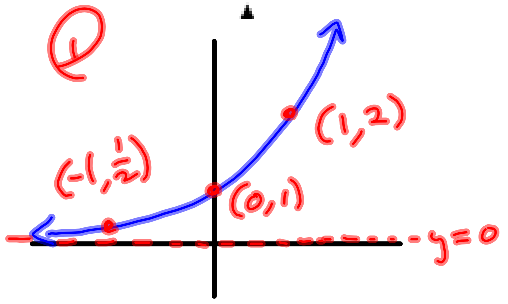
If time permits, I'll do another video on completing the square and those first two Writing Projects, only less clumsily. :o|

Test 4 – Spring, 2011 Name_______________________

Covers Chapter 4, plus 7.1, 7.3 NO GRAPHING CALCULATOR!!!

1. (20 pts) Starting with $f(x) = 2^x$, sketch the graph of $g(x) = 4\cdot 2^{x+1} - 5$ in 4 steps (counting $f(x) = 2^x$ as the first step). Use $x = -1$, $x = 0$, and $x = 1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Your final graph should also show the y-intercept and, for 5 bonus points, the x-intercept.

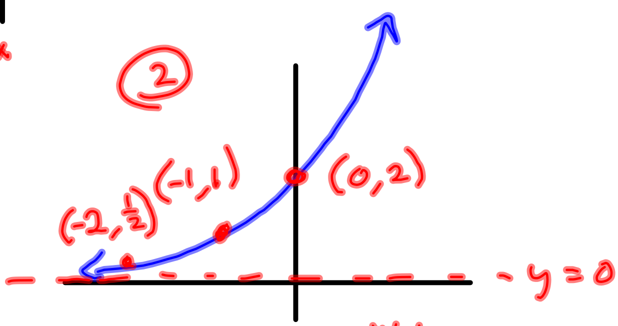
1



$f(x) = 2^x$

$2^{-1} = \frac{1}{2}$

2

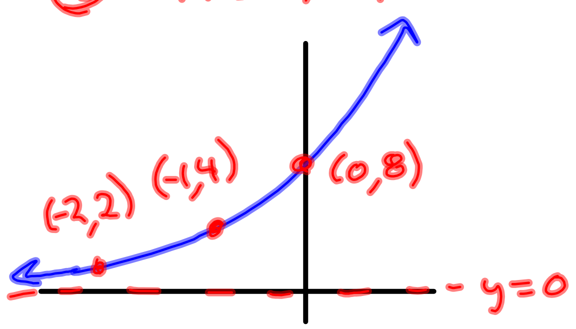


$f(x+1) = 2^{x+1}$

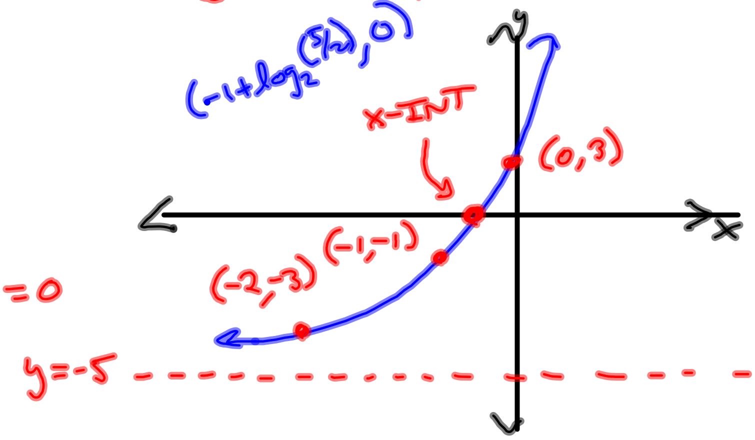
$2^x \longrightarrow 2^{x+1} \longrightarrow 4\cdot 2^{x+1} \longrightarrow 4\cdot 2^{x+1} - 5$

$f(x)$ ① $\quad f(x+1)$ ② left 1 $\quad 4f(x+1)$ ③ 4 times y $\quad 4f(x+1)-5$ ④ Down 5

③ $4f(x+1) = 4 \cdot 2^{x+1}$



④ $4f(x+1) - 5 = 4 \cdot 2^{x+1} - 5$



X-INT: $y = 0$

$$4 \cdot 2^{x+1} - 5 = 0$$

$$4 \cdot 2^{x+1} = 5$$

$$2^{x+1} = \frac{5}{4}$$

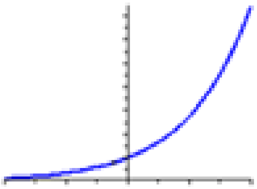
$$\log_2\left(2^{x+1}\right) = \log_2\left(\frac{5}{2}\right)$$

$$x+1 = \log_2\left(\frac{5}{2}\right)$$

$$x = -1 + \log_2\left(\frac{5}{2}\right) \Longrightarrow$$

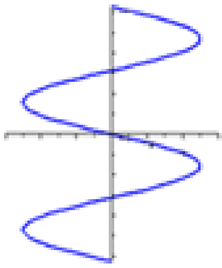
x-int:
$$\left(-1+\log_2\left(\frac{5}{2}\right), 0\right)$$

2. (10 pts) Determine which of the following are functions and whether they are one-to-one. So indicate by writing "Yes" or "No" in the appropriate spaces.



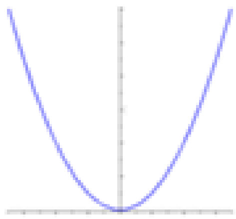
Function? Yes

1-to-1? Yes



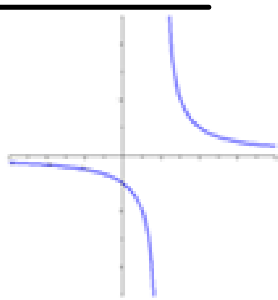
Function? No

1-to-1? DNA



Function? Yes

1-to-1? No



Function? Yes

1-to-1? Yes

3. For $f(x)=\sqrt{x-1}$ and $g(x)=2x+1$, determine the following composite functions, *simplify them*, and state their domains:

a. (5 pts) $(f\circ g)(x) = f(g(x)) = \sqrt{g(x)-1} = \sqrt{2x+1-1}$

$= \sqrt{2x}$

$\mathcal{D}(f\circ g) = \{x \mid x\in\mathcal{D}(g) \text{ and } g(x)\in\mathcal{D}(f)\}$

$\mathcal{D}(g) = \mathbb{R} = (-\infty,\infty)$

$g(x) = 2x+1$

$f(x)$ wants $x-1\ge 0$

$x\ge 1$

$f(g(x))$ wants $g(x)-1\ge 0$

$2x+1-1\ge 0$

$2x\ge 0$

$\boxed{x\ge 0}$

$x\ge 0 \Rightarrow$

$\boxed{\mathcal{D} = [0,\infty)}$

$= \{x \mid x\ge 0\}$

b. (5 pts) $(g\circ f)(x)$

$= g(f(x)) = 2f(x)+1 = \boxed{2\sqrt{x-1}+1} = g(f(x))$

$x\in\mathcal{D}(f)$ and $f(x)\in \boxed{\mathcal{D}(g)=\mathbb{R}}$

Just need $f(x)$ to be happy

No restriction

Need $x-1\ge 0$

$x\ge 1$

So $\boxed{\mathcal{D}(g\circ f) = \{x \mid x\ge 1\} = [1,\infty)}$

4. (5 pts) What is the domain of $g(x) = \ln(-2x+3)$?

ln(STUFF) Needs STUFF > 0

Need $-2x+3>0$

$-2x>-3 \Longrightarrow x<\frac{-3}{-2}=\frac{3}{2}$

$\frac{-2x}{-2}<\frac{-3}{-3}$

$\boxed{D=\{x \mid x<\tfrac{3}{2}\}=(-\infty,\tfrac{3}{2})}$

5. (5 pts) What is the domain of $\ln\left(\frac{(x-2)^2}{(x-3)(x+1)^3}\right)$? (This is like a Chapter 3 question!)

Need $\frac{(x-2)^2}{(x-3)(x+1)^3}>0$

Test: $x=4$

$\frac{(4-2)^2}{(4-3)(4+1)^3}=\frac{2^2}{1(5)^3}>0$

+ − − +

>0 | <0 | <0 | >0

−1 2 3

$\boxed{(-\infty,-1)\cup(3,\infty)=D}$

6. (5 pts) Let $f(x) = 5^{x+1} - 4$. Find $f^{-1}(x)$.

$y = 5^{x+1} - 4$

$x = 5^{y+1} - 4$ & solve for y.

$5^{y+1} - 4 = x$

$5^{y+1} = x + 4$

$y + 1 = \log_5(x+4)$

$y = \boxed{\log_5(x+4) - 1 = f^{-1}(x)}$

7. (5 pts) Find functions f and g so that $f \circ g = H$, given that $H(x) = \ln(x^2 - 1)$.

$f(g(x)) = H(x)$

Let $g(x) = x^2 - 1$

$f(x) = \ln(x)$

$(f \circ g)(x) = \ln(x^2 - 1)$ ✓

8. (5 pts) Evaluate $\log_2(96) - \log_2(3)$ *without a calculator* !!

$$\log_2(96) - \log_2(3)$$
$$= \log_2\left(\frac{96}{3}\right) = \log_2(32) =$$
$$= \log_2(2^5) = \boxed{5}$$

$2 \,|\, 32$, $2 \,|\, 16$, $2 \,|\, 8$, $2 \,|\, 4$, 2

9. (5 pts) Solve *without a calculator*: $5^{x-1} = 3^x$. All I want is a symbolic answer and the symbolic manipulations you perform to *get* there. For full credit, your answer should involve a logarithm or two in it.

$$5^{x-1} = 3^x$$
$$\log_5(5^{x-1}) = \log_5(3^x)$$
$$x - 1 = x \cdot \log_5(3)$$

$$x - x\log_5(3) - 1 = 0$$
$$x - x\log_5(3) = 1$$
$$x(1 - \log_5(3)) = 1$$
$$x = \frac{1}{1 - \log_5(3)}$$

$$x - 1 = \log_5(3)\, x$$
$$x - 1 = Ax \text{, where } A = \log_5(3)$$
$$x - Ax = 1$$
$$x(1 - A) = 1$$
$$x = \frac{1}{1-A} = \boxed{\frac{1}{1 - \log_5(3)}}$$

$$5^{x-1} = 3^x$$
$$\ln(5^{x-1}) = \ln(3^x)$$
$$(x-1)\ln(5) = (x)\ln(3)$$

Let
$$A = \ln(5)$$
$$B = \ln(3)$$

$$(x-1)A = (x)B = Bx$$
$$Ax - A = Bx$$
$$Ax - Bx = A$$

$$x(A - B) = A$$
$$x = \frac{A}{A-B}$$
$$= \boxed{\frac{\ln(5)}{\ln(5) - \ln(3)} = x}$$

Nov 10-8:59 AM

10. (5 pts) Write the following as the logarithm of a single expression. Assume that variables represent positive numbers. $3\log_5(x+7)-2\log_5(x-7)+\log_5 9$

$$= \log_5\left((x+7)^3\right) - \log_5\left((x-7)^2\right) + \log_5(9)$$

$$= \log\left(\frac{(x+7)^3 \cdot 9}{(x-7)^2}\right)$$

11. (10 pts) Solve: $\ln(x-4)+\ln(x+1)=\ln(6)$ for x.

$$e^{\ln((x-4)(x+1))} = e^{\ln(6)}$$

$$(x-4)(x+1) = 6$$

$$x^2+x-4x-4 = x^2-3x-4 = 6$$

$$x^2-3x-10=0$$

$$(x-5)(x+2)=0$$

$$x=5 \text{ OR } \cancel{x=-2}$$

Final answer.

$\ln(x-4)$ needs $x>4$

$\ln(x+1)$ needs $x>-1$

12. Find the geometric sums:

a. (5 pts) $2+\frac{2}{3}+\frac{2}{9}+\cdots+\frac{2}{2187}$ (Be careful finding your a, r, and n in $a\cdot r^{n-1}$)

$\sum_{n=1}^{\infty} a\cdot r^{n-1}$

$a=2$

$\frac{2}{3}=2\cdot\frac{1}{3} \Rightarrow r=\frac{1}{3}$

$\frac{2}{3}\cdot\frac{1}{3}=\frac{2}{9}$

$\sum_{n=1}^{?} 2\cdot\left(\frac{1}{3}\right)^{n-1}$

3 | 2187
3 | 729
3 | 243
3 | 81
3 | 27
3 | 9
3

$\frac{2}{2187}=2\left(\frac{1}{3}\right)^{7}$

And the 7 is my "n-1" So $n=8$ = "Top n"

$\sum_{n=1}^{8} 2\left(\frac{1}{3}\right)^{n-1} = \frac{2\left(1-\left(\frac{1}{3}\right)^{8}\right)}{1-\frac{1}{3}} = \frac{2\left(1-\left(\frac{1}{3}\right)^{8}\right)}{\frac{2}{3}} = \frac{\cancel{2}\left(1-\left(\frac{1}{3}\right)^{8}\right)\cdot 3}{\cancel{2}} = 3\cdot\left(1-\left(\frac{1}{3}\right)^{8}\right) \approx \boxed{2.99954}$

$\frac{a(1-r^n)}{1-r}$

TI-30II

b. (5 pts) $\sum_{i=1}^{\infty} 7\cdot\left(\frac{3}{5}\right)^{i-1}$ $\quad \frac{3}{5}=r<1$

$\sum_{k=1}^{\infty} 7\cdot\left(\frac{3}{5}\right)^{k-1} = \frac{7}{1-\frac{3}{5}} = \frac{7}{\frac{4}{5}} = \frac{\frac{7}{1}}{\frac{4}{5}} = \frac{7}{1}\cdot\frac{5}{4} = \boxed{\frac{35}{4}}$

$\frac{a}{1-r}$

```
+Y1(13/4)+Y1(15/
4)
          22024.20363
Ans/2
          11012.10182
3*(1-(1/3)^8)
          2.999542753
```

13. The half-life of carbon-14 is (approximately) 5500 years.

a. (5 pts) Derive the exponential decay model $A(t) = A_0 e^{-kt}$. ~~The trick, here,~~ is to find the decay rate, k, based on the half-life given.

$$\boxed{A(t) = A_0 e^{-kt}}$$

$\frac{1}{2}$-life is 5500 yrs

$$A(5500) = A_0 e^{-k \cdot 5500} = \boxed{A_0 e^{-5500k} = \frac{1}{2} A_0}$$

$$e^{-5500k} = \frac{1}{2}$$

$$-5500k = \ln(1/2)$$

$$\boxed{k = \frac{\ln(1/2)}{-5500}} = \frac{\ln(1) - \ln(2)}{-5500} = \boxed{\frac{\ln(2)}{5500} = k}$$

b. (5 pts) Use your model from above to predict the age of an ancient fire pit, if a charcoal sample from the pit contains 20% of its original carbon-14. For ease of solving this problem, you may want to just use a symbolic k until the last step. Round your final answer to the nearest year.

```
ln(.2)*5500/ln(2
)
         -12770.60452
-Ans
          12770.60452
```

$$\cancel{A_0} e^{-kt} = .20 \cancel{A_0}$$

$$e^{-kt} = .2$$

$$-kt = \ln(.2)$$

$$t = \frac{\ln(.2)}{-k} = \frac{\ln(.2)}{-\frac{\ln(2)}{5500}} = -\ln(.2) \cdot \frac{5500}{\ln(2)}$$

$\approx$ 12771 years old.