1. (10 pts) Is the relation $f = \{(\underline{3},-7),(\underline{4},-1),(\underline{2},5),(\underline{9},-1)\}$ a function? Explain in words.

$\boxed{\text{Yes}}$ No repetitions in 1st coordinate.

2. (5 pts) What's the domain of f? $\mathcal{D} = \{3,4,2,9\}$

3. (5 pts) What's the range of f? $\mathcal{R} = \{-7,-1,5\}$

$y=-1$ shows up twice (Not 1-to-1)

4. Let $f(x)=\dfrac{x+7}{x-2}$ and $g(x)=\sqrt{x+1}$.

a. (5 pts) What is the domain of f?

Need $x-2 \neq 0$

$\mathcal{D} = \{x \mid x \neq 2\}$

$= (-\infty,2)\cup(2,\infty)$

b. (5 pts) What is the domain of g?

Need $x+1 \geq 0$

$\mathcal{D} = \{x \mid x \geq -1\}$

$= [-1,\infty)$

c. Determine the following functions. You don't need to simplify. In fact, I recommend you do not.

$$\frac{x+7}{x-2} = f(x) \quad , \quad g(x) = \sqrt{x+1}$$

i) (5 pts) $f+g$ $= (f+g)(x) = f(x)+g(x)$

$$= \frac{x+7}{x-2} - \sqrt{x+1}$$

ii) (5 pts) $f \circ g$ $= (f \circ g)(x) = f(g(x))$

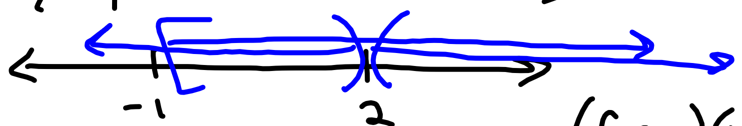
$$= \frac{g(x)+7}{g(x)-2} = \frac{\sqrt{x+1}+7}{\sqrt{x+1}-2}$$

d. (5 pts) What is the domain of $f+g$? State your final answer in interval notation.

$$\mathcal{D}(f+g) = \mathcal{D}(f-g) = \mathcal{D}(fg) = \mathcal{D}(f) \underset{\text{And}}{\cap} \mathcal{D}(g)$$

$$= \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g)\}$$

$$= \{x \mid x \neq 2 \text{ and } x \geq -1\}$$



$$= [-1,2) \cup (2,\infty)$$

$(fg)(x)$

$fg =$

$(f \circ g)(x)$

$f(x)g(x) =$

$$f(g(x)) \neq (fg)(x)$$

e. (5 pts) What is the domain of $f \circ g$? State your final answer in interval notation.

$$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$$

$$= \{x \mid x \geq -1 \text{ and } \sqrt{x+1} \neq 2\}$$

$$= \{x \mid x \geq -1 \text{ and } x \neq 3\}$$

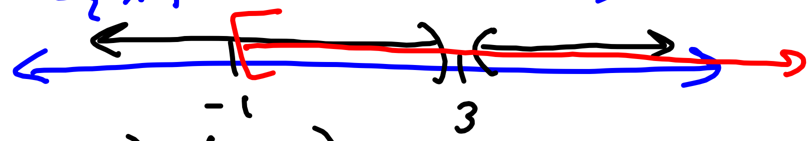
$\sqrt{x+1} \neq 2$

$(\sqrt{x+1})^2 \neq 2^2$

$x+1 \neq 4$

$x \neq 3$

=



$$= [-1,3) \cup (3,\infty)$$

5. (5 pts) Simplify the difference quotient, $\frac{f(x+h)-f(x)}{h}$, for $f(x)=x^2-5x$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-5(x+h)-[x^2-5x]}{h}$$

$$= \frac{x^2+2xh+h^2-5x-5h-x^2+5x}{h}$$

$$= \frac{2xh+h^2-5h}{h} = \frac{\cancel{h}(2x+h-5)}{\cancel{h}}$$

$$= \boxed{2x+h-5}$$

Bonus Pass to the limit, as $h \to 0$, on your answer to the above, so you can show me some calculus.

$$\xrightarrow{h \to 0} \boxed{2x-5}$$

6. (5 pts) Explain to me why the equation $x^2 + y^2 = 49$ does *not* define y as a function of x.

$y^2 = 49 - x^2$

$\sqrt{y^2} = \sqrt{49-x^2}$

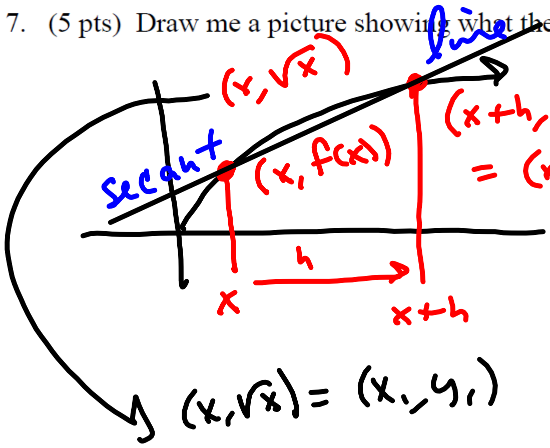
$|y| = \sqrt{49-x^2}$

$y = \pm\sqrt{49-x^2}$ → Here's the problem 2 y-values for x-values

$x = 0 \rightarrow y = \pm\sqrt{49} = \pm 7$ Two y-values?!

Need ONE y-value!

7. (5 pts) Draw me a picture showing what the difference quotient represents for the function $f(x) = \sqrt{x}$.



$(x+h, f(x+h)) = (x+h, \sqrt{x+h}) = (x_2, y_2)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$

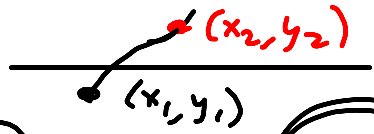
= slope of secant line between two points on the curve.

8. (10 pts) Answer one of the following:

a. Show that $f(x)=\frac{2}{3}x-7$ is one-to-one, algebraically.

b. If $f(x)=x^2-6x+7$, for $x\geq 3$, what is $f^{-1}(x)$?.

a. $x_1 \neq x_2 \Rightarrow y_1 \neq y_2$



$$\boxed{\begin{aligned} y_1 = y_2 &\Rightarrow x_1 = x_2 \\ \tfrac{2}{3}x_1 - 7 &= \tfrac{2}{3}x_2 - 7 \\ \tfrac{2}{3}x_1 &= \tfrac{2}{3}x_2 \\ x_1 &= x_2 \end{aligned}}$$

Yes 1-to-1.

b. x^2-6x+7 $x\geq 3$

$f^{-1}(x)$:

$y^2-6y+7=x$

$y^2-6y+3^2-9+7=x$

$(y-3)^2-2=x$

$(y-3)^2=x+2$

$y-3=\pm\sqrt{x+2}$

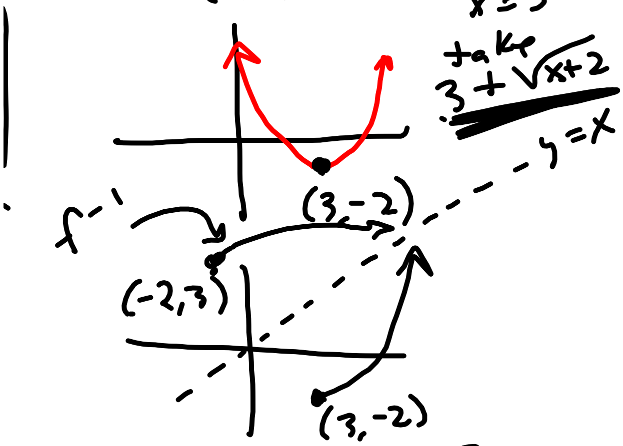
$y=3\pm\sqrt{x+2}$

$x\leq -3$ take $3-\sqrt{x+2}$

$y=x^2-6x+7$

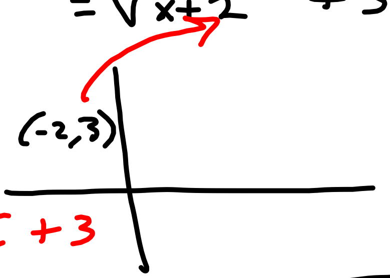
$=(x-3)^2-2$

$x\geq 3$ take $3+\sqrt{x+2}$



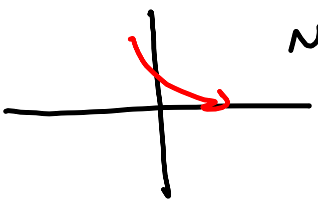
$3+\sqrt{x+2}$

$=\sqrt{x+2}\ +3$



what about

$y=3-\sqrt{x+2}=-\sqrt{x+2}+3$



Not mirror image of f across $y=x$

$y=\sqrt{x+2}\ +3$ is $f^{-1}(x)$

Sep 22-1:26 PM

9. (10 pts) Show that $f(x)=\dfrac{x+3}{x-1}$ is its own inverse. In other words, show that in this example, f and f^{-1} are the same, exact function! There are two ways to accomplish this:
   1. By finding f^{-1}, directly.
   2. By the definition of f^{-1}.

$$f \circ f = f(f(x)) = \frac{\frac{x+3}{x-1}+3}{\frac{x+3}{x-1}-1} = \frac{\frac{x+3+3(x-1)}{x-1}}{\frac{x+3-1(x-1)}{x-1}}$$

$$= \frac{\frac{x+3+3x-3}{x-1}}{\frac{x+3-x+1}{x-1}} = \frac{\frac{4x}{x-1}}{\frac{4}{x-1}} = \frac{4x}{x-1}\cdot\frac{x-1}{4} = x$$

So f^{-1} <u>is</u> f

$f(f(x)) = x$

$$\frac{x+3}{x-1} = y \qquad \frac{y+3}{y-1} = x$$

$y+3 = (y-1)x$

$y+3 = xy - x$

$y - xy = -x-3$

$y(1-x) = -x-3$

$$y = \frac{-x-3}{1-x} = \frac{x+3}{x-1} = f(x) = f^{-1}(x)$$

yeppers.

10. (10 pts) Suppose y varies jointly with m_1 and m_2, and inversely with the square of r. Write an equation describing this situation.

$$y = k\frac{m_1 m_2}{r^2}$$

Other things I want to see from students on Test 2, that, miraculously, don't appear on this old test from 2013:

1. Graphs and properties of basic functions
2. Shifting, stretching, reflecting shrinking basic functions to transform them into new functions. Graphing these!
3. Completing the square skills for re-writing

$$ax^2 + bx + c \text{ as } a(x-h)^2 + k$$

The Cheat:

$$h = \frac{-b}{2a} \text{ and } k = f(h) = f\left(-\frac{b}{2a}\right)$$

4. Horizontal reflections, stretches and shrinkages are NOT adequately covered by your textbook.

One of my favorite type-graphs is something like $g(x) = -2\sqrt{5-10x} + 3$