Final Test

Name_____
NO GRAPHING CALCULATORS!!!

Do your own work. SHOW your work. When in doubt about how stupid I am, assume the worst.

1. Solve the following inequalities. Give the solution in set-builder notation and interval notation.

a. (10 pts) $|8x - 5| \geq 3$

$8x - 5 \geq 3 \quad$ OR $\quad 8x - 5 \leq -3$

$8x \geq 8 \quad$ OR $\quad 8x \leq 2$

$\{x \mid x \geq 1 \quad$ OR $\quad x \leq \tfrac{1}{4}\}$

$= (-\infty, \tfrac{1}{4}] \cup [1, \infty)$

b. (10 pts) $|8x - 5| \leq 3$

$8x - 5 \leq 3 \quad$ AND $\quad 8x - 5 \geq -3$

$8x \leq 8 \quad$ AND $\quad 8x \geq 2$

$\{x \mid x \leq 1 \quad$ AND $\quad x \geq \tfrac{1}{4}\}$

$= [\tfrac{1}{4}, 1]$

c. (5 pts) $|x - 2| \leq -4$

Never! $\emptyset$

d. (5 pts) $|x - 2| > -4$

Always $(-\infty, \infty)$

2. Solve the following equations, by any method, other than copying a classmate.

a. (20 pts) $4x^2 - 12x + 7 = 0$

$a = 4, b = -12, c = 7$

$b^2 - 4ac = (-12)^2 - 4(4)(7)$

$= 144 - 112 = 32$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{12 \pm \sqrt{32}}{2(4)} = \dfrac{12 \pm \sqrt{32}}{8} = \dfrac{12 \pm 4\sqrt{2}}{8}$

$$= \boxed{\dfrac{3 \pm \sqrt{2}}{2}}$$

b. (10 pts) $\dfrac{21}{x+5} - \dfrac{5}{x-3} = 8$

$\dfrac{21(x-3) - 5(x+5)}{LCD} = \dfrac{8(x+5)(x-3)}{LCD}$

$21x - 63 - 5x - 25 = 8(x^2 + 2x - 15)$

$16x - 88 = 8x^2 + 16x - 120$

$8x^2 - 32 = 0$

$8(x^2 - 4) = 0$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$\boxed{x = \pm 2}$

3. (10 pts) $\sqrt{19 - 2x} = x - 2$

$19 - 2x = x^2 - 4x + 4$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = \boxed{-3, 5} \longrightarrow \text{Nope}$

$\boxed{x = 5}$

4. (10 pts) Simplify $\dfrac{\left(6x^{-3}y^4\right)^3}{\left(10x^5y^{-1}\right)^4}$. Assume all variables represent nonnegative real numbers. Your final

answer should contain only positive exponents.

$$= \frac{6^3 x^{-9} y^{12}}{10^4 x^{20} y^{-4}} = \frac{2^3 3^3 \, x^{-9+20} \, y^{12+4}}{2^4 \cdot 5^4}$$

$$= \frac{3^3}{2 \cdot 5^4} \, x^{-29} y^{16} = \frac{3^3 \, y^{16}}{2 \cdot 5^4 \, x^{29}} = \frac{27 \quad y^{16}}{1250 \quad x^{29}}$$

5. (20 pts) Simplify $\dfrac{12 \pm \sqrt{32}}{8} = \dfrac{12 \pm 4\sqrt{2}}{8} = \dfrac{4\left(3 \pm \sqrt{2}\right)}{8}$

$$= \boxed{\frac{3 \pm \sqrt{2}}{2}}$$

$$2\,\underline{)\,32}$$
$$2\,\underline{)\,16}$$
$$2\,\underline{)\,8}$$
$$2\,\underline{)\,4}$$
$$2$$

6. (20 pts) Answer one of the following.
   a. A man bought a book on sale at a 20% discount. If he paid $37.00 at the register (after the discount!), what was the original price of the book (before the discount!).
   b. John can paint a room in 7 hours. Jane can paint a room in 5 hours. How long does it take them, working together?

(a) $x - .2x = 37$

$.8x = 37$

$x = \dfrac{37}{.8}$

$\approx \boxed{\$46.25}$

$x = $ price of book before discount.

(b) $\dfrac{1}{7} + \dfrac{1}{5} = \dfrac{1}{t}$

$5t + 7t = 35$

$12t = 35$

$t = \dfrac{35}{12} \approx \boxed{2.9167}$ hrs

$t = $ time it takes to paint room, in hours

7. (10 pts) Solve $x^2 - 8x - 19 = 0$ by completing the square.
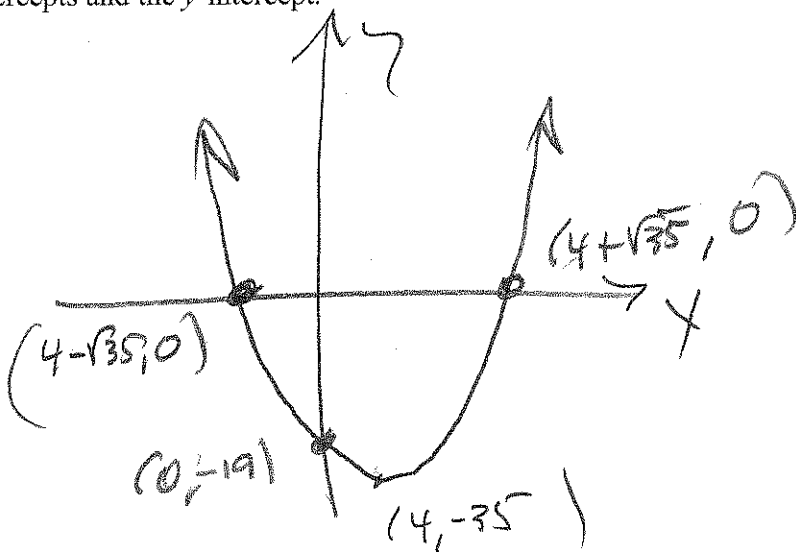
$$x^2 - 8x = 19$$

$$x^2 - 8x + 4^2 = 19 + 16$$

$$(x-4)^2 = 35$$

$$x - 4 = \pm\sqrt{35}$$

$$\boxed{x = 4 \pm \sqrt{35}}$$

8. (10 pts) Re-write the function $f(x) = x^2 - 8x - 19$ in the form $a(x-h)^2 + k$ and sketch the graph. Your graph should include Vertex, both x-intercepts and the y-intercept.

$$x^2 - 8x - 19$$

$$= x^2 - 8x + 4^2 - 16 - 19$$

$$= (x-4)^2 - 35$$

$(4-\sqrt{35}, 0)$

$(4+\sqrt{35}, 0)$

$(0, -19)$

$(4, -35)$

9. (10 pts) Use synthetic division to determine $f(3)$ for $f(x) = 3x^4 - 2x^3 - 5x^2 + 7x - 11$. In other words, divide $f(x)$ by $x - 2$, using synthetic division, and interpret!

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3 | 3  -2  -5   7   -11
  |     9  21  48   165
  ------------------------
    3   7  16  55 | 154 = f(3)
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10. (10 pts) Use long division to determine the quotient and remainder for $\dfrac{3x^4 - 5x^2 + 7x - 11}{x^2 - 2}$. Write your

final answer in the form of $Dividend = (Divisor)(Quotient) + Remainder$

$$
\begin{array}{r}
3x^2 + 1 \\
x^2 - 2 \enclose{longdiv}{3x^4 + 0x^3 - 5x^2 + 7x - 11} \\
-(3x^4 \qquad - 6x^2) \\
\hline
x^2 + 7x - 11 \\
-(x^2 \qquad - 2) \\
\hline
7x - 9
\end{array}
$$

$3x^4 - 5x^2 + 7x - 11 = (x^2 - 2)(3x^2 + 1) + 7x - 9$

11. (10 pts) Sketch the graph of the system of inequalities: $\begin{array}{l} 3x - 2y \ge 6 \\ 7x + 3y \ge 21 \\ x \ge 0 \\ y \ge 0 \end{array}$. Clearly label the "Good Stuff!"

$3x - 2y \ge 6$

x	y
0	-3
2	0

$(0,0)$ BAD

$7x + 3y \ge 21$

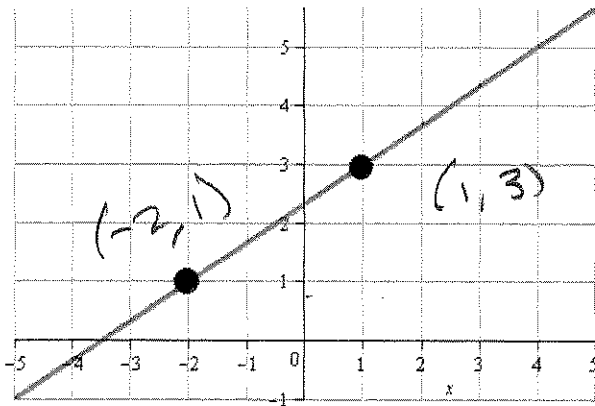| x	y
0	3
7	0

$(0,0)$ BAD
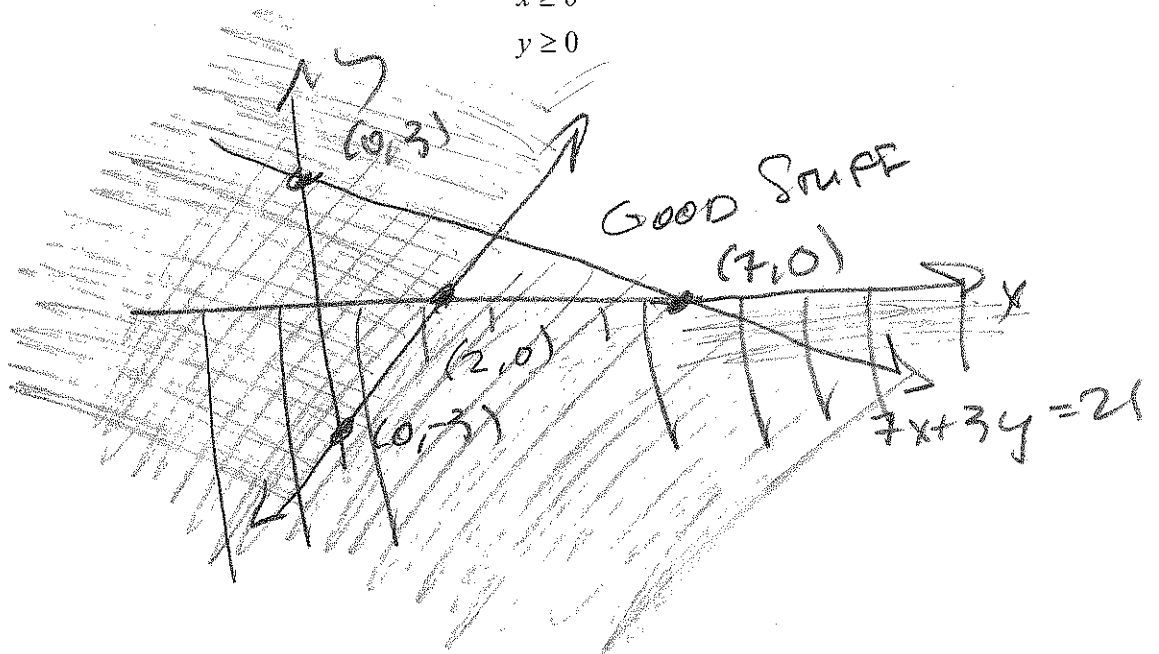
12. (20 pts) Write an equation for the line shown in the picture:

$$m = \frac{3-1}{1-(-2)} = \frac{2}{3}$$

$$y = \frac{2}{3}(x-1) + 3$$

OR

$$y = \frac{2}{3}(x+2) + 1$$

$$= \frac{2}{3}x + \frac{4}{3} + \frac{3}{3}$$

$$= \frac{2}{3}x + \frac{7}{3}$$

$$\approx .667x + 2.667$$

Answer up to 2 bonus questions for up to 20 points. I will grade the first 3 you do work on, unless you tell me to omit them.

1. (10 pts) Consider the equation $ax^2 + bx + c = 0$. Write the discriminant.
2. (10 pts) What's the solution of the equation $ax^2 + bx + c = 0$?
3. (10 pts) Solve $3x^2 - 2x + 5 = 0$ by completing the square.
4. (10 pts) Write $\dfrac{3+2i}{8-7i}$ in the standard form $a+bi$.

① $b^2 - 4ac$

② $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

④ $\left(\dfrac{3+2i}{8-7i}\right)\left(\dfrac{8+7i}{8+7i}\right)$

$= \dfrac{24 + 21i + 16i + 14i^2}{64 + 49} = \dfrac{10}{113} + \dfrac{37}{113}i$

③ $3\left(x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2\right) = -5 + \frac{1}{3}$

$3\left(x - \frac{1}{3}\right)^2 = \frac{-14}{3}$

$x - \frac{1}{3} = \pm\sqrt{\frac{-14}{9}} = \pm\frac{\sqrt{14}}{3}i \longrightarrow x = \dfrac{1 \pm \sqrt{14}\,i}{3}$