§5.1, 6.2 Linear Systems.
↳ Matrices, Elimination (Gaussian)

2 variables: where do the lines meet, if they meet?

$y=3x+2$ is a line — parallel or not parallel

$3x-y=-2$ same line

Expressing one variable in terms of one other, free variable. That means line. One degree of freedom.

$$\begin{aligned} 3x + 2y &= 6 \\ y &= 7 \end{aligned}$$

is triangular. Solve by back-substitution.

No 'x' in 2nd Equation.

$y=7 \longrightarrow 3x+2(7)=6$

$3x+14=6$

$3x=-8$

$x=-\frac{8}{3}$

Unique Sol'n

$(x,y) \in \{(-8/3, 7)\}$ is sol'n set statement, for $(x,y) = (-8/3, 7)$

No Sol'n

$3x + 2y = 6$
$-15x - 10y = 13$

Goal is triangular system

Use the 3x to kill the -5x

R1 $\quad 3x + 2y = 6$

5R1+R2 $\quad 0 = 43$

5R1 $\quad 15x + 10y = 30$
R2 $\quad -15x - 10y = 13$

$0 = 43$

FALSE!

We arrived at an absurd/false conclusion. This is because our solution technique is based on the ASSUMPTION that there is a solution.

Reasoning from a faulty premise leads to absurdity!

$\boxed{\text{No Solution}}$

3 Variables Do the planes share point(s)

(x, y, z) ?

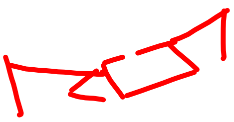
Unique solution - Corner where 2 walls & ceiling meet.

Many Solutions - 3 planes meet in a line

OR

(All 3 planes same) → 3 planes meet in a plane

No Solutions - More ways this can happen



Parallel Planes.

Other - Pup tent.

Combos.

Nov 14-8:11 AM

Solve the system:

$9x - 3y + z = 84$

$x + y + z = 8$

$4x + 2y + z = 24$

$x + y + z = 8$

$9x - 3y + z = 84$

$4x + 2y + z = 24$

$$\begin{array}{lrcl} -9R1 & -9x - 9y - 9z &=& -72 \\ R2 & 9x - 3y + z &=& 84 \\ \hline & -12y - 8z &=& 12 \end{array}$$

$\boxed{* -\frac{1}{4} \quad 3y + 2z = -3}$

optional move

R1	$x + y + z = 8$
$-9R1 + R2$*	$3y + 2z = -3$
$-4R1 + R3$	$-2y - 3z = -8$

$$\begin{array}{lrcl} -4R1 & -4x - 4y - 4z &=& -32 \\ R3 & 4x + 2y + z &=& 24 \\ \hline & -2y - 3z &=& -8 \end{array}$$

$x + y + z = 8$

$3y + 2z = -3$

$-2y - 3z = -8$

R1 $x + y + z = 8$

R2 $3y + 2z = -3$

2R2+3R3 $z = 6$ *

Back-Substitution:

$3y + 2(6) = -3$

$3y + 12 = -3$

$3y = -15$

$y = -5$

$x - 5 + 6 = 8$

$x + 1 = 8$

$x = 7$

2R2 $6y + 4z = -6$

3R3 $-6y - 9z = -24$

$-5z = -30$

* $z = 6$

optional

$\boxed{(x, y, z) \in \{(7, -5, 6)\}}$

ordered triple

3-tuple

Nov 14-8:30 AM

$f := x \rightarrow 7 \cdot x^2 - 5 \cdot x + 6$

$x \rightarrow 7\,x^2 - 5\,x + 6$

$f(1)$ How I built the previous question.

8

$7(1)^2 - 5(1) + 6 = 8$

$f(1) = 8$

$f(-3)$

84

$f(-3) = 84$

$f(2)$

$f(2) = 24$

24

Want to find a, b, c for

$f(x) = ax^2 + bx + c$

System:

$f(1) = 8$: $a + b + c = 8$

$f(-3) = 84$: $9a - 3b + c = 84$

$f(2) = 24$: $4a + 2b + c = 24$

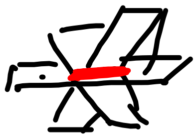
$x + y + z = 8$

$9x - 3y + z = 84$

$4x + 2y + z = 24$

That was 1 solution

Many Solutions : 3 planes meet in a line



Paddle wheel

$$-2x + 3y + 10z = 25$$
$$2x + 5y + 22z = 39$$
$$x + 3y + 14z = 23$$

$$x + 3y + 14z = 23$$
$$-2x + 3y + 10z = 25$$
$$2x + 5y + 22z = 39$$

R1 $\quad x + 3y + 14z = 23$

2R1+R2 $\quad 9y + 38z = 71$

−2R1+R3 $\quad -y - 6z = -7$

2R1 $\quad 2x + 6y + 28z = 46$

R2 $\quad -2x + 3y + 10z = 25$

$9y + 38z = 71$

−2R1 $\quad -2x - 6y - 28z = -46$

R3 $\quad 2x + 5y + 22z = 39$

$-y - 6z = -7$

R1 $\quad x + 3y + 14z = 23$

R2 $\quad 9y + 38z = 71$

R2+9R3 $\quad z = -\frac{1}{2}$

etc.

Screwup Teacher

R2 $\quad 9y + 38z = 71$

9R3 $\quad -9y - 54z = -63$

$-16z = 8$

$z = -\frac{1}{2}$

This was supposed to be the last one, darn it!

$2x + 3y + 10z = 25$
$2x + 5y + 22z = 39$
$x + 3y + 14z = 23$

S6.1 way

$$\left[\begin{array}{ccc|c} 2 & 3 & 10 & 25 \\ 2 & 5 & 22 & 39 \\ 1 & 3 & 14 & 23 \end{array}\right]$$

Augmented Coefficient matrix

$$\left[\begin{array}{ccc|c} 1 & 3 & 14 & 23 \\ 2 & 3 & 10 & 25 \\ 2 & 5 & 22 & 39 \end{array}\right]$$

$$\begin{array}{r} -46 \\ 25 \\ \hline -21 \end{array} \qquad \begin{array}{r} -46 \\ +39 \\ \hline -7 \end{array}$$

$$\begin{array}{l} R1 \\ -2R1+R2 \\ -2R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 14 & 23 \\ 0 & -3 & -18 & -21 \\ 0 & -1 & -6 & -7 \end{array}\right]$$

$$\begin{array}{l} R1 \\ R2 \\ R2-3R3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 14 & 23 \\ 0 & -3 & -18 & -21 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{rrr} -3 & -18 & -21 \\ 3 & +18 & +21 \\ \hline 0 & 0 & 0 \end{array}$$

Is triangular. One Row disappeared.

$-3y - 18z = -21$
$y + 6z = 7$
$y = -6z + 7$

$x + 3y + 14z = 23$
$x + 3(-6z+7) + 14z = 23$
$x - 18z + 21 + 14z = 23$
$x - 4z + 21 = 23$
$x = 4z + 2$

$(x, y, z) \in \{(4z+2, -6z+7, z) \mid z \in \mathbb{R}\}$

This is a line in space!
One free variable
one-dimensional object.