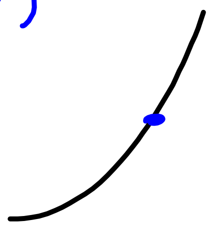
Today
Change of Base
Practical Apps
Games with Log Props.

My calculator won't do $\log_3(77)$



$$\log_3(77) = x$$

$$3^{\log_3(77)} = 3^x$$

$$77 = 3^x$$

$$\ln(77) = \ln(3^x)$$

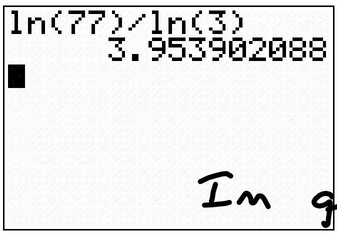
$$\ln(77) = x\ln(3)$$

$$\ln(77) = \ln(3)\,x$$

$$\ln(3)\,x = \ln(77)$$

$$x = \frac{\ln(77)}{\ln(3)}$$

I can do dis with my calculator.



≈ 3.9539, to 4 places.

$\ln(x) = \log_e(x)$

$\log(x) = \log_{10}(x)$

$$\log_4(4^7) =$$

$$= 7\log_4(4)$$

$$= 7$$

In general,

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

$$\log_3(77) = \frac{\ln(77)}{\ln(3)} = \frac{\log(77)}{\log(3)} = \frac{\log_5(77)}{\log_5(3)}$$

APR = 8% compounded, \$5000

① 6 yrs, ② 8 yrs, ③ 5 yrs, ④ 20 yrs, 321 days.

Compounded daily

① $5000\left(1+\frac{.08}{365}\right)^{365(6)}$

② $5000\left(1+\frac{.08}{365}\right)^{365(8)}$

⋮

④ $\boxed{5000\left(1+\frac{.08}{365}\right)^{(20(365)+321)}}$

20 yrs + 321 days

$=(20\text{ yrs})\left(\frac{365\text{ days}}{1\text{ yr}}\right)+321\text{ days}$ = # of periods.

n = # of periods = $\boxed{mt}$

m = # of periods / yr = 365

t = …… years

321 days = $(321\text{ days})\left(\frac{1\text{ yr}}{365\text{ days}}\right)$ is now in years.

$\boxed{5000\left(1+\frac{.08}{365}\right)^{365\left(20+\frac{321}{365}\right)}}$

```
5000(1+.08/365)^
(20*365+321)
        26565.43809
5000(1+.08/365)^
(365(20+321/365)
        26565.43809
```

$$\left(\frac{1}{2}\right)^x = 512$$

$$\left(\frac{1}{2}\right)^x = 2^9$$

without a calculator, the trick is to write both sides as a power of the same base.

$$2\,|\,512$$
$$2\,|\,256$$
$$2\,|\,128$$
$$2\,|\,64$$
$$2\,|\,32$$
$$2\,|\,16$$
$$2\,|\,8$$
$$2\,|\,4$$
$$2$$

$$\left(2^{-1}\right)^x = 2^9$$

$$2^{-x} = 2^9$$

$$-x = 9$$

$$x = -9$$

$$\left(\frac{1}{2}\right)^x = \left(\left(\frac{1}{2}\right)^{-1}\right)^9$$

$$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-9}$$

$$x = -9$$

Nov 3-8:13 AM

$$\left(\frac{1}{2}\right)^x = 512$$

$$\ln\left(\left(\frac{1}{2}\right)^x\right) = \ln(512)$$

$$x \ln\left(\frac{1}{2}\right) = \ln(512)$$

$$\left(\ln\left(\frac{1}{2}\right)\right) x = \ln(512)$$

$$x = \frac{\ln(512)}{\ln\left(\frac{1}{2}\right)} = -9$$

$$\left(\frac{1}{2}\right)^x = 512$$

$$\log_{1/2}\left(\left(\frac{1}{2}\right)^x\right) = \log_{1/2}(512)$$

$$x = \log_{1/2}(512) = \frac{\ln(512)}{\ln(1/2)} = -9$$

convert by change of base

$\frac{1}{2}$ - life questions.

$$P = P(t) = P_0 e^{kt}$$

Think of it as function of time.

P_0 = Initial amount

K = Decay / Growth rate.

$\frac{1}{2}$-life of Millsium is 50 yrs.

A sample of Millsium has only 10% of naturally occurring radioactive Millsium remaining. How old is the sample?

Common Sense:

50 yrs	$1/2 = .5$
100 yrs	$1/4 = .25$
150 yrs	$1/8 = .125$
200 yrs	$1/16 = .0625$

Method:
use known $\frac{1}{2}$-life to find K.
Use K to find age.

$$P = P(t) = P_0 e^{kt}$$

Nov 3-8:21 AM

$\frac{1}{2}$-life is 50 yrs.

In 50 yrs, we have $\frac{1}{2}$ of what we started with.

$P(50) = P_0 e^{k \cdot 50} = P_0 e^{50k} = \frac{1}{2} P_0$

Solve for k to get the model.

$e^{50k} = \frac{1}{2}$

$\ln(e^{50k}) = \ln(1/2)$

$50k = \ln(1/2)$

$k = \frac{\ln(1/2)}{50} = \frac{-\ln(2)}{50}$

$\ln(1/2) = -\ln(2)$

$\ln(2^{-1}) = -1\ln(2)$

$\boxed{k = -\frac{\ln(2)}{50}} \approx -.0138629$

How old's the millsium?

$P(t) = P_0 e^{kt} = .1 P_0$ Solve this for t.

```
n(.1)*50/ln(2)
     -166.0964047
ln(.1)*50/ln(2)

      166.0964047
ln(2)/50
     -.0138629436
```

$e^{kt} = .1$

$\ln(e^{kt}) = \ln(.1)$

$kt = \ln(.1)$

$t = \frac{\ln(.1)}{k}$

$= \frac{\ln(.1)}{-\frac{\ln(2)}{50}} = \frac{-\ln(.1)}{\ln(2)} \cdot 50$

≈ 166.10 yrs.

```
      26565.43809
ln(.1)*50/ln(2)
     -166.0964047
-ln(.1)*50/ln(2)

      166.0964047
```

Nov 3-9:01 AM

Questions

Props of Logs.

Simplify. Write with positive exponents

$$\frac{(x^2y^{-3}z^5)^7}{(x^{2/3}y^4z^{-6})^3} = \frac{x^{14}y^{-21}z^{35}}{x^2y^{12}z^{-18}} = \frac{x^{12}z^{53}}{y^{33}}$$

Write as a sum/difference of simpler logs.

$$\log\left((x^2y^{-3}z^5)^7\right) = \log(x^{14}y^{-21}z^{35})$$

or $7\log(x^2y^{-3}z^5) =$

$$7\left[\log(x^2) + \log(y^{-3}) + \log(z^5)\right]$$

$$= 7\left[2\log(x) + (-3)\log(y) + 5\log(z)\right]$$

$$= 14\log(x) - 21\log(y) + 35\log(z)$$

$\log(z^6) = 6\log(z)$

$\log(xy) = \log(x) + \log(y)$

Next time, clinic on exponential and logarithmic equations.

$$\boxed{\log(x+1) + \log(x-1) = \log(2x)}$$

$$\log_{10}((x+1)(x-1)) = \log_{10}(2x)$$

$$(x+1)(x-1) = 2x$$

$$x^2 - 1 = 2x$$

$$x^2 - 2x - 1 = 0$$

So, that's it.