§4.3, 4.4 questions up
§4.3 solns up
§4.3, due Wed.

Today: Establish the number 'e', for Euler.

Simple interest
r = annual rate of interest (APR)
t = time in years
P = Principal
A = Accumulated Amount.
I = Interest accumulated.

$I = Pr$ for one year.

฿100 @ 10% apr accumulates to ฿110 after one year.

$A = P + Prt$ gives simple interest after t years.
Accumulated Amount.

$P(1+rt)$
$\uparrow \frac{P}{P}$ $\uparrow \frac{Prt}{P}$

Compound Interest

Collect interest, plow it back into principle.

m = the # of compoundings per year.

Compounded annually : m = 1

$100 @ 10%, compounded annually

1 yr : $100 + (100)(.1) = \$110 = P + Pr(1)$

2 yrs : $110 + (110)(.1) = \$110 + \$11 = \$121$

3 yrs : $121 + (121)(.1) = \$121 + \$12.10 = \$133.10$ Emma.

1 period $P + Pr$

2 period $(P+Pr) + (P+Pr)r$

$= P(1+r) + P(1+r)r$

$= P(1+r)(1+r)$

$= P(1+r)^2$ 2 years, compounded annually.

3 periods $P(1+r)^2 + P(1+r)^2 r$

$P(1+r)^2[1+r] = P(1+r)^3$

what if $m = 12$? use $r = .1$

Then let $i = \frac{r}{m} = \frac{r}{12} = \frac{.1}{12}$

1 mo. $P(1+\frac{r}{m}) = P(1+i) = P(1+\frac{.1}{12})$

2 mo. $P(1+\frac{r}{m}) + P(1+\frac{r}{m})\frac{r}{m}$

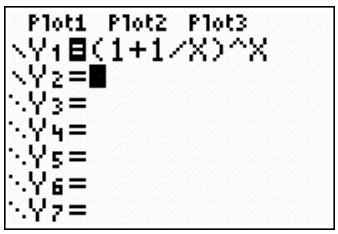
$= P(1+\frac{.1}{12})^2$

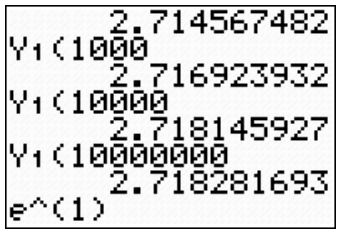
⋮

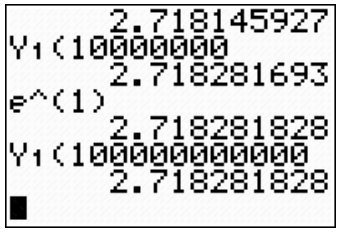
n months $A = P(1+\frac{.1}{12})^n$

$= 100(1+\frac{.1}{12})^n$

t years: $n = mt = 12t$



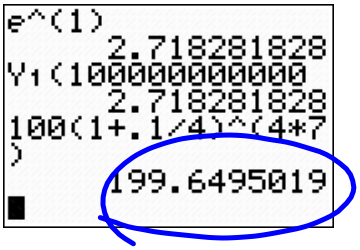




$P(1+\frac{r}{m})^{mt}$ bridges Pe^{rt} as $m \to \infty$.

How much does \$100 compounded quarterly accumulate to in 7 years? (10% APR)

$100\left(1+\frac{.1}{4}\right)^{(4)(7)}$



Close to double!

How long 'til it triples?

Same question parameters

Principal has tripled.

Accumulation is 3 times what we started with.

$$A = 3P$$

$$P\left(1+\frac{r}{m}\right)^{mt} = 3P$$

$$\left(1+\frac{.1}{4}\right)^{4t} = 3$$

Solve this for t & we have tripling rate.

Suppose tripling time is 20 years.
What's interest rate?
Assume compounded daily, with 365-day year.

$$A = 3P$$

$$P\left(1+\frac{r}{365}\right)^{(365)(20)} = 3P$$

Same question, continuous compounding.

$$A = 3P$$

$$Pe^{rt} = 3P$$

$$Pe^{30r} = 3P$$ Solve for r.

The half-life of Carbon-14 is 5730 yrs. What's the decay rate?

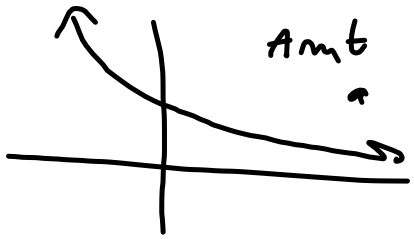
$A = \frac{1}{2}P$

$Pe^{rt} = \frac{1}{2}P$ ← $\frac{1}{2}$-life equation

$Pe^{5730r} = \frac{1}{2}P$ ← Solve for r to get decay rate.

Decay? $r < 0$

Pe^{rt} when r is negative:



Amt of C-14 in a sample.