$(x-(3-7i))(x-(3+7i))$

$= (x-3+7i)(x-3-7i)$

3-7i is a
3+7i conjugate pairs.

$= x^2-3x-7ix-3x+9+21i+7ix-21i-49i^2$

$= x^2-6x+9+49$

$= x^2-6x+58$

$(a-b)(a+b)=a^2-b^2$

$\boxed{(a-bi)(a+bi)=a^2+b^2}$

$(3-2i)(5+i)$

$x^2-(3+7i)x-(3-7i)x+(3-7i)(3+7i)$

$x^2-3x-7ix-3x+7ix+9+49$

etc.

Multiply and Simplify

$(2\cdot x - 3 + I)\cdot(2\cdot x - 3 - I)$

Write a polynomial, in factored form, of minimal degree that has the following zeros, with the corresponding multiplicities.

$x = 13,\ m = 3;$

$x = -5,\ m = 1;$

$x = -7 + 2i$, $m = 1$

$(x-13)^3(x+5)(x-(-7+2i))$

Write a polynomial, in factored form, of minimal degree that has the following zeros, with the corresponding multiplicities, and real coefficients. Why is your answer to this one different than the one before?

$x = 13,\ m = 3;$

$x = -5,\ m = 1;$

$x = -7 + 2i$

$(x-13)^3(x+5)(x-(-7+2i))(x-(-7-2i))$

Conjugate Pairs Theorem

Write a polynomial, in factored form, of minimal degree that has the following zeros, with the corresponding multiplicities, and real coefficients. Also make it have a leading coefficient of 377.

$x = 13,\ m = 3;$

$x = -5,\ m = 1;$

$x = -7 + 2i$

$377(x-13)^3(x+5)(x-(-7+2i))(x-(-7-2i))$

What does Descartes have to say about this polynomial?

$2x^5 - 8x^4 + x^3 + 19x^2 - 7x - 15 = f(x)$
1 2 3

3 or 1 positive zeros

$f(-x) = -2x^5 - 8x^4 - x^3 + 19x^2 + 7x - 15$
1 2

2 or 0 negative zeros.

$(-x)^4 = x^4$

$(-x)^{315} = -x^{315}$

$(-1)^{315} = -1$

$(-1)^{578} = 1$

Which would you check first?

Check the positives first.

No guarantee their are ANY negative roots.

What are the possible rational zeros of this polynomial?

$2x^5 - 8x^4 + x^3 + 19x^2 - 7x - 15$

Rational Zeros Theorem

p's $15 = 3 \cdot 5$

q's 2

$\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 5, \pm \frac{5}{2}, \pm 15, \pm \frac{15}{2}$

Show that $x = 7$ is an upper bound on real zeros of the polynomial.

$$2x^5 - 8x^4 + x^3 + 19x^2 - 7x - 15$$

43
7

7 |	2	−8	1	19	−7	−15
		14	42	Big	Bigger	Huge
	2	6	43	BIG	BIGGISH	Huge

All positives

Proof:

$f(x) = (x-7)(2x^4 + 6x^3 + 43x^2 + \text{Big}\, x + \text{Bigger}) + \text{Huge}.$

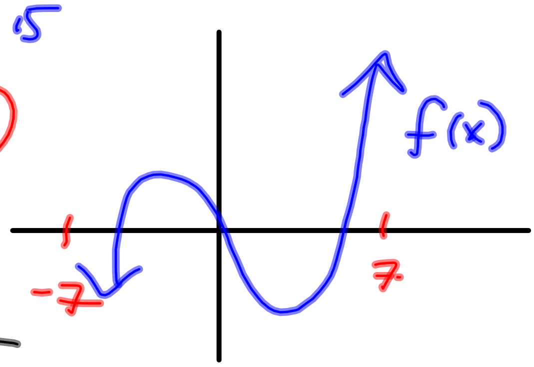
is positive, if x is bigger than 7.

Show that −7 is a lower bound on real zeros.

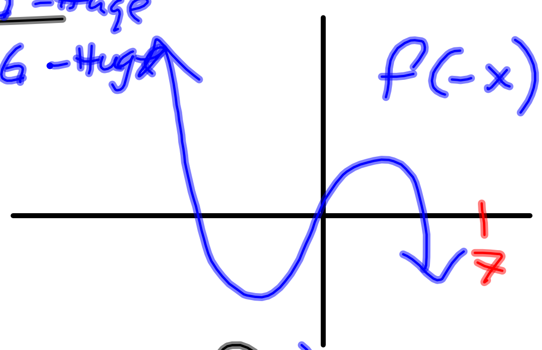
$f(-x) = -2x^5 - 8x^4 - x^3 + 19x^2 + 7x - 15$

0

Want to show $x=7$ is bound on $f(-x)$!



7 |	−2	−8	−1	+19	7	−15
		−14	−144	−Big	−BIG	−Huge
	−2	−22	−145	−BIG	BIG	−Huge



This says

$(x-7)(-2x^4 - 22x^3 - 145x^2 - \text{BIG}\, x - \text{BIG}) - \text{HUGE}$

So anything bigger than 7 is going make this negative. never zero.

$x = -7$ is a lower bound on $f(x)$, b/c

$x = 7$ " an upper " " $f(-x)$.

Find all real zeros of the polynomial and factor over the real numbers.

$$2x^5 - 8x^4 + x^3 + 19x^2 - 7x - 15$$

$\ldots \pm 1, \pm 3$

$$\begin{array}{r|rrrrrr} 3 & 2 & -8 & 1 & 19 & -7 & -15 \\ & & 6 & -6 & -15 & 12 & 15 \\ \hline -1 & 2 & -2 & -5 & 4 & 5 & 0 \\ & & -2 & 4 & 1 & -5 & \\ \hline -1 & 2 & -4 & -1 & 5 & 0 & \\ & & -2 & 6 & -5 & & \\ \hline & 2 & -6 & 5 & 0 & & \end{array}$$

Yes!

Niiiice.

Sweeeeeeeet!

$a=2, b=-6, c=5$

$b^2-4ac = (-6)^2-4(2)(5) = 36-40 = -4$

No real zeros for $2x^2-6x+5$ is irreducible over the reals.

$(x+1)^2(x-3)(2x^2-6x+5)$ is factored over $\mathbb{R}$.

Find *all* zeros of the polynomial and factor over the complex numbers.

$2x^5 - 8x^4 + x^3 + 19x^2 - 7x - 15$

$(x+1)^2(x-3)(2x^2-6x+5)$ is factored over $\mathbb{R}$.

Break this down, to finish.

$a=2, b=-6, c=5$

$b^2-4ac = (-6)^2-4(2)(5) = 36-40 = -4$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{6 \pm \sqrt{-4}}{2(2)} = \frac{6 \pm 2i}{4} = \frac{\cancel{2}(3 \pm i)}{\cancel{2}(2)}$

$= \frac{3 \pm i}{2}$

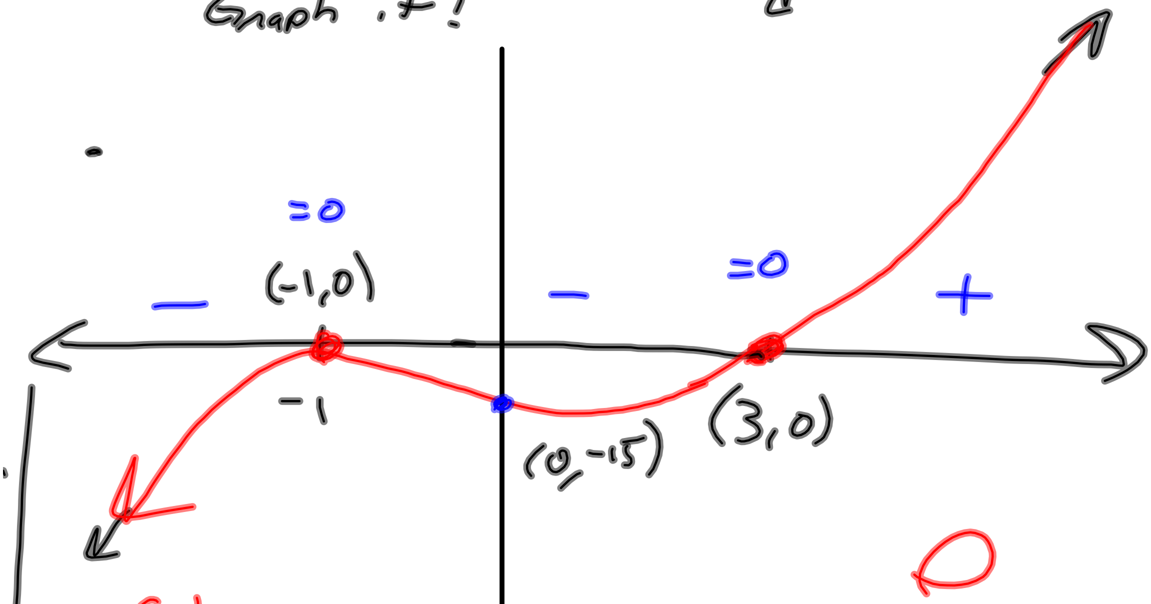
So, $f(x) =$

$2(x+1)^2(x-3)\left(x-\left(\frac{3+i}{2}\right)\right)\left(x-\left(\frac{3-i}{2}\right)\right)$

$2x^5$. — — — .

$(x+1)^2(x-3)(2x^2-6x+5)$

Graph it!



Solve

$(x+1)^2(x-3)(2x^2-6x+5) \leq 0$

$(-\infty,-1] \cup [-1,3] = (-\infty,3]$

$\ldots < 0$

$(-\infty,-1) \cup (-1,3)$

$\ldots\ldots > 0$

$(3,\infty)$

$\ldots\ldots \geq 0$

$\{-1\} \cup [3,\infty)$