$f(x)=\sqrt{x-2}$ $\qquad g(x)=\dfrac{x-7}{x+5}$

FROM Test 2

5pts

$(f\circ g)(x)=\sqrt{\dfrac{x-7}{x+5}-2}$

$f(g(x))$

$\mathcal{D}(f)=\{x \mid x\ge 2\}=[2,\infty)$, $\mathcal{D}(g)=\{x \mid x\ne -5\}=(-\infty,-5)\cup(-5,\infty)$

$\mathcal{D}(f\circ g)=\{x \mid x\in\mathcal{D}(g) \text{ and } g(x)\in\mathcal{D}(f)\}$

$=\{x \mid x\ne -5 \text{ and } \dfrac{x-7}{x+5}\ge 2\}$

$\dfrac{x-7}{x+5}\ge 2$ is a C3 skill.

$x-7\ge 2(x+5)$ $\qquad -3x\ge 6$

Bad move! $\qquad x\le -2$

$x+5<0$ oR $x+5>0$?

$\dfrac{x-7}{x+5}\ge \dfrac{2}{1}\cdot\dfrac{(x+5)}{x+5}=\dfrac{2x+10}{x+5}$

$\dfrac{x-7}{x+5}\ge\dfrac{2x+10}{x+5}$

$\dfrac{x-7}{x+5}-\dfrac{2x+10}{x+5}\ge 0$

$\dfrac{x-7-(2x+10)}{x+5}\ge 0$

$\dfrac{x-7-2x-10}{x+5}\ge 0$

1 $\boxed{\dfrac{-x-17}{x+5}\ge 0}$ Sign pattern time.

Recall $\underbrace{-(x+17)^1(x+5)^1}\ge 0$

Is a parabola that opens down.

$-(x)(x)=-x^2$ controls end behavior

$x=-17,-5$ are Key.

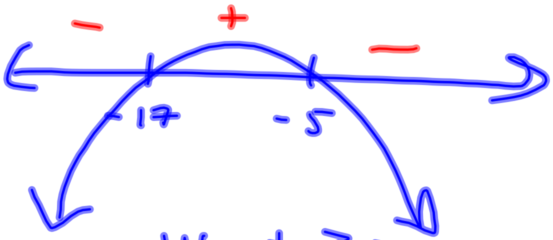
$(-x-17)(x+5)$

$(-x)(x) = -x^2$ is End behavior ↙ ↘

Check: Test $x=0$

$(-0-17)(0+5)$

$= (-17)(5)$



Want ≥ 0

$[-17, -5]$ solves $-(x+17)(x+5) \geq 0$

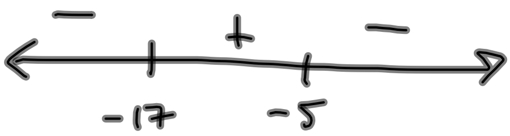
What solves $-\dfrac{(x+17)}{x+5} \geq 0$?

End Behavior:

$-\dfrac{x}{x} = -1$

Same Sign Pattern!

Only difference here, is can't let $x=-5$



It exploded

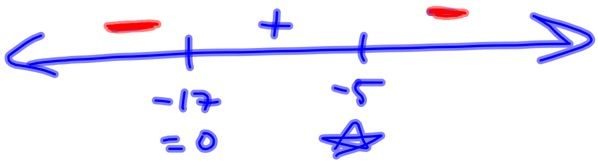
$$\boxed{\mathcal{S} = [-17, -5)}$$

$$\frac{x-7}{x+5} \ge 2$$

$$\frac{x-7-2(x+5)}{x+5} \ge 0$$

$$\frac{-x-17}{x+5} \ge 0$$

$-\frac{x}{x} = \boxed{-1 = y} =$ HORIZONTAL ASYMPTOTE.



want ≥ 0

$\therefore$ +

$[-17, -5)$

#3 §3.2.

$-x^3+x^2-4x+9$

Check $x+3$ is factor?

$$\begin{array}{r|rrrr} -3 & -1 & 1 & -4 & 9 \\ & & 3 & -12 & 48 \\ \hline & -1 & 4 & -16 & 57 \end{array}$$

QUOTIENT $\qquad \dfrac{57}{x+3}$ $\qquad \dfrac{\text{REMAINDER}}{\text{Divisor}}$

$$f(x) = \frac{-x^3+x^2-4x+9}{x+3} = -x^2+4x-16 + \frac{57}{x+3}$$

In 3.4, this work shows the end behavior of $f(x)$: $-x^2+4x-16$, because $\frac{57}{x+3}$ vanishes when you get far away from $x=-3$.

This $y=-x^2+4x-16$ is called an oblique asymptote. They occur when the numerator is of greater degree than the denominator. We find O.A. by division. It's the quotient.

O.A.

V.A.

H.A.

Graphs of Rational Functions and Inequalities involving same, are very closely linked.

Rational Function $R(x) = \frac{P(x)}{Q(x)}$, where $P(x)$, $Q(x)$ are polynomials.

① PROPER $\frac{1}{x}$, $\frac{x^2-2x}{x^3+1}$, $\frac{x-7}{x^2-5x+2}$

H.A. $y=0$ $y=0$ $y=0$

② IMPROPER $\frac{x-1}{x+2}$, $\frac{x^2-5x}{3x^2+2}$, $\frac{2x^3+5x}{x^3-17}$

$y=1$ $y=\frac{1}{3}$ $y=2$

H.A. !

$\frac{2x^2-5x+1}{x+1}$

Find OBLIQUE Asymptote

$$\begin{array}{r|rrr} -1 & 2 & -5 & 1 \\ & & -2 & 7 \\ \hline & 2 & -7 & 8 \\ & x' & c & r \end{array}$$

So this one is actually

$2x-7+\frac{8}{x+1}$

§3.5 Graphs.

Graph it!

$$f(x) = \frac{4x}{x^2-3x-4}$$

① Domain (Factor top & bottom)

② Asymptotes

$$= \frac{4x}{(x-4)(x+1)}$$

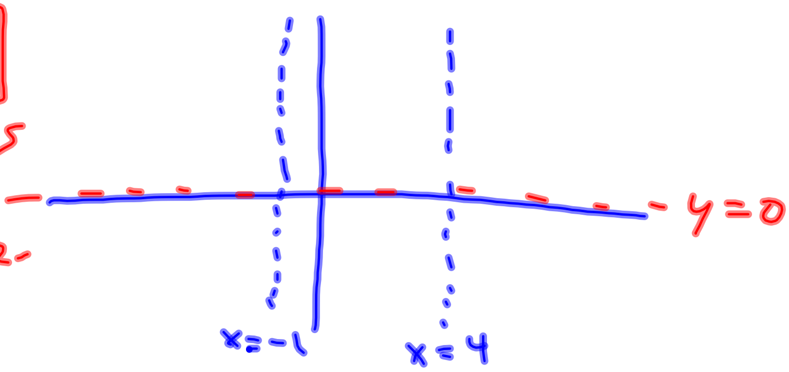
$\mathcal{D} = \mathbb{R} \setminus \{-1, 4\}$

$\boxed{\text{V.A. } x=-1,\ x=4}$ vertical Asymptotes. V.A. Almost always work this way.

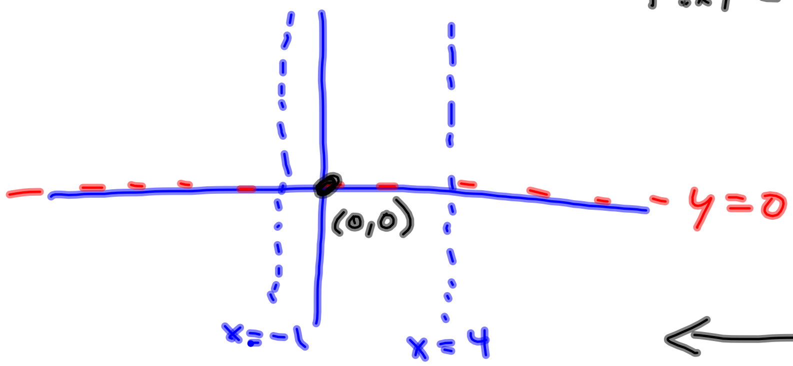
H.A. $\boxed{y=0}$

O.A.

Asymptotes Frame the picture.



$$f(x) = \frac{4x}{(x-4)(x+1)} = \frac{4x}{x^2-3x-4}$$



Intercepts:

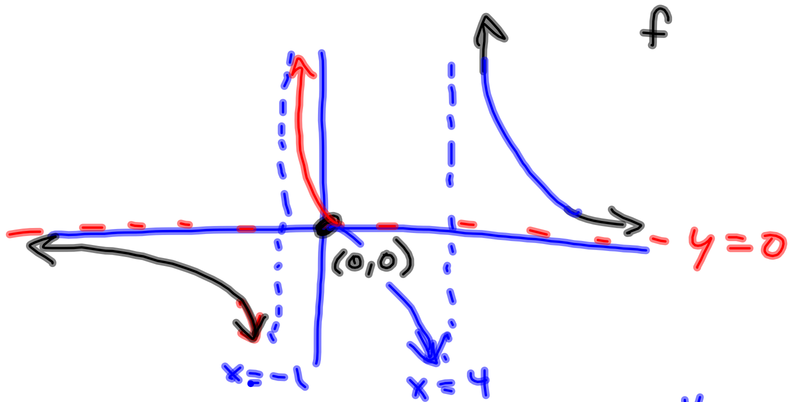
$f(0) = \frac{0}{-4} = 0 \rightsquigarrow (0,0)$ is y-int.

x-int: $f(x) = 0$

$$\frac{4x}{x^2-3x-4} = 0$$

$$4x = 0$$

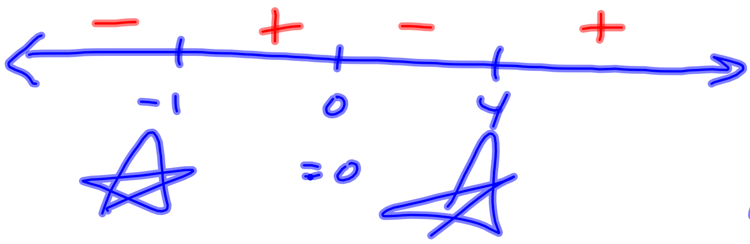
$$x = 0 \rightsquigarrow (0,0)$$



PROPER

$x^1 = (x-0)^1$

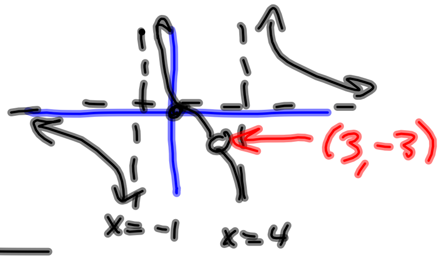
Analyze Sign Pattern $\frac{4x}{(x-4)(x+1)^1} = \frac{4x}{x^2-3x-4}$



Sign Pattern for Proper is a little tougher, because $y=0$ is no guide to +/−

$$\frac{4x}{(x-4)(x+1)}$$

$$-\frac{4(3)}{(\ \ 4)} = -\frac{12}{4} = -3$$



$$\frac{4x(x-3)}{(x-4)(x+1)(x-3)} = \frac{4x^2-12x}{x^3-6x^2+5x+12}$$

$$(x-3)(x^2-3x-4) = \begin{array}{r} x^3-3x^2-4x \\ -3x^2+9x+12 \\ \hline x^3-6x^2+5x+12 \end{array}$$

The exception to V.A. Rule.

$D = \mathbb{R} \setminus \{-1, 3, 4\}$

V.A. $x=-1$, $x=4$

HOLE: $x=3$