3.2 Is $x+3$ a factor of
$f(x) = -x^3 + x^2 - 4x + 9$?

$$\begin{array}{r|rrrr} -3 & -1 & 1 & -4 & 9 \\ & & 3 & -12 & 48 \\ \hline & -1 & 4 & -16 & 57 = f(-3) \end{array}$$

No!

This work says $f(x) = (x+3)(-x^2+4x-16) + 57$

Goal is to split any polynomial into linear factors.

$(x-c)$

Factor Theorem : Remainder Theorem when remainder is zero. find a zero? found a factor. Polynomial is depressed. :(

#8 Is $x+3$ a factor? If so, factor "completely over the complex #s".

split into linear factors.

$x^3 + 4x^2 + x - 6 = f(x)$

$$\begin{array}{r|rrrr} -3 & 1 & 4 & 1 & -6 \\ & & -3 & -3 & 6 \\ \hline & 1 & 1 & -2 & 0 \\ & x^2 & x & c & r \end{array}$$

Sweet!

So $f(x) =$

$(x+3)\underline{(x^2+x-2)}$

I split off $x+3$

x^2+x-2 is the depressed polynomial.

$x^2 + x = 2$

$x^2 + x + \left(\frac{1}{2}\right)^2 = 2 + \frac{1}{4} = \frac{9}{4}$

$\left(x+\frac{1}{2}\right)^2 = \frac{9}{4}$

$x + \frac{1}{2} = \pm \frac{3}{2}$

$x = \frac{-1 \pm 3}{2}$ → $\frac{2}{2} = 1$, $\frac{-4}{2} = -2$

$f(x) = (x+3)(x-1)(x+2)$ is now split into linear (1^{st} degree) factors.

Rational Zeros

$f(x) = (3x-2)(2x+5) = 0$

$\Rightarrow x = \frac{2}{3}, -\frac{5}{2}$

Notice $f(x) = 6x^2 + 11x - 10$ &

$x = \frac{2}{3}$ is a zero

2 is a factor of -10

3 " " " " 6

-5 " " " " -10

2 " " " " 6

If $\frac{p}{q}$ is a zero of $f(x)$ then p is a factor of a_0 & q is a factor of a_n

$a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$

Possible Rational Zeros:

$2x^4 - 5x^2 - 7x + 11$

$\frac{p}{q}$: $\pm 1, \pm \frac{1}{2}, \pm 11, \pm \frac{11}{2}$ are all the guesses.

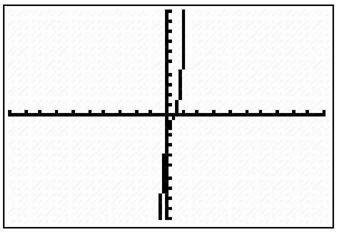
Find all real & nonreal zeros

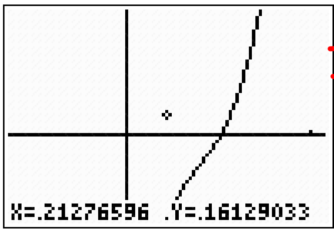
$18x^3 - 21x^2 + 10x - 2$ $\quad 2, \quad 2 \mid 18, \quad 3 \mid 9, \quad 3$

$\frac{p}{q}: \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{1}{9}, \pm \frac{1}{18}$

$\pm 2, \cancel{\pm \frac{2}{2}}, \pm \frac{2}{3}, \cancel{\pm \frac{2}{6}}, \pm \frac{2}{9}, \cancel{\pm \frac{2}{18}}$

For homework, use grapher!





I guess $x = \frac{1}{2}$ will work

$\frac{1}{2}$	18	−21	10	−2
		9	−6	2
	18	−12	4	0

sweet!

$(x - \frac{1}{2})(18x^2 - 12x + 4)$

Split it!

$18x^2 - 12x + 4 = 0$

iff $9x^2 - 6x + 2 = 0$

$a = 9, b = -6, c = 2$

$b^2 - 4ac = (-6)^2 - 4(9)(2)$

$= 36 - 72 = -36$

$\sqrt{-36} = 6i$

Error in my solutions.

$x = \frac{6 \pm 6i}{2(9)} = \frac{3 \pm 3i}{9} = \frac{1 \pm i}{3}$

This work says $f(x) = 18(x - \frac{1}{2})(x - (\frac{1+i}{3}))(x - (\frac{1-i}{3}))$

NOTE $18x^2 - 12x + 4$ is irreducible over the real numbers. No real zeros. Can't factor any further without going "nonreal

#20-22

Re-write $\frac{\text{Dividend}}{\text{Divisor}}$ as Quotient + $\frac{\text{Remainder}}{\text{Divisor}}$

Read ahead
watch videos

$$\frac{2x+1}{x+2} = 2 + \frac{-3}{x+2}$$

$$\begin{array}{r|rr} -2 & 2 & 1 \\ & & -4 \\ \hline & 2 & -3 \\ & c & r \end{array}$$

$$\frac{2x+1}{x-2} = 2 + \frac{5}{x-2}$$

$$\begin{array}{r|rr} 2 & 2 & 1 \\ & & 4 \\ \hline & 2 & 5 \end{array}$$

$6x^3 + 25x^2 - 24x + 5 = f(x)$ Split it.

p: 5
q: 6

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6},$
$\pm 5, \pm \frac{5}{2}, \pm \frac{5}{3}, \pm \frac{5}{6}$

Change 3.3
#13

$$\begin{array}{r|rrrr} 1 & 6 & 25 & -24 & 5 \\ & & 6 & 31 & \text{New p} \\ \hline & 6 & 31 & 6 & \end{array}$$

$$\begin{array}{r|rrrr} -1 & 6 & 25 & -24 & 5 \\ & & -6 & -19 & \\ \hline & 6 & 19 & \text{New p} & \end{array}$$

$$\begin{array}{r|rrrr} 5 & 6 & 25 & -24 & 5 \\ & & 30 & \text{New p} & \\ \hline & 6 & 55 & & \end{array}$$

Gud!

$$\begin{array}{r|rrrr} -5 & 6 & 25 & -24 & 5 \\ & & -30 & 25 & -5 \\ \hline & 6 & -5 & 1 & \end{array}$$

$(x+5)(6x^2 - 5x + 1)$

$a = 6, b = -5, c = 1$

$b^2 - 4ac = (-5)^2 - 4(6)(1)$
$= 25 - 24$
$= 1$

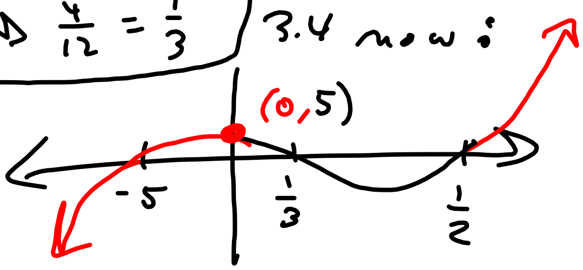
$\sqrt{\ } = 1$

$x = \frac{5 \pm 1}{2(6)} = \frac{6}{12} = \frac{1}{2}, \quad \frac{4}{12} = \frac{1}{3}$

$f(x) = 6(x+5)^1(x-\frac{1}{2})^1(x-\frac{1}{3})^1$
$= (x+5)(2x-1)(3x-1)$

3.4 now:

Sketch:



$6x^3$

E.B. ↙…↗

Descartes' Rule of Signs.

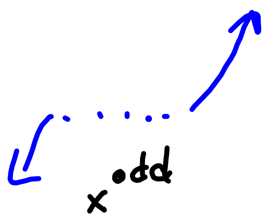
$f(x) = 6x^3 + 25x^2 \overset{1}{-} 24x \overset{2}{+} 5$

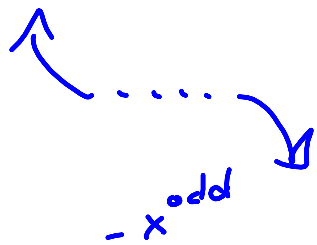
2 or 0 positive zeros

$f(-x) = -6x^3 \overset{1}{+} 25x^2 + 24x + 5$

$\boxed{\text{1 negative zero}}$

Any odd-degree poly has at least ONE real zero





§3.2, 3.3 Due Monday

§3.4, 3.5 Due Wednesday

Test Friday/Monday

$$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{(2a)^2} = \frac{b^2}{2^2a^2}$$

$$= \frac{b^2}{4a^2}$$

Oct 17-8:56 AM