① a $\mathcal{D}(f) \cap \mathcal{D}(g) = \{x \mid x \in \mathcal{D}(f) \text{ AND } x \in \mathcal{D}(g)\}$
$= \mathcal{D}(f+g) = \mathcal{D}(f-g) = \mathcal{D}(f \cdot g)$

b $\mathcal{D}(\frac{f}{g}) = \mathcal{D}(f) \cap \mathcal{D}(g) \cap \{x \mid g(x) \neq 0\}$
$= \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \text{ and } g(x) \neq 0\}$

c $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

② a 1-to-1 : $x_1 \neq x_2 \Rightarrow y_2 \neq y_1$ $\frac{x_1 - 1}{x_1 + 5} = \frac{x_2 - 1}{x_2 + 5}$

To Show 1-to-1 Assume $y_1 = y_2$
Prove $x_1 = x_2$

b f^{-1} : g is the inverse of f if $g \circ f = f \circ g = x$
$f(g(x)) = g(f(x)) = x$

c. $\mathcal{D}(f^{-1}) = \mathcal{R}(f)$
$\mathcal{R}(f) = \mathcal{D}(f^{-1})$

③ $y = \frac{K x z^3}{\sqrt{w}}$, for some Constant, K.

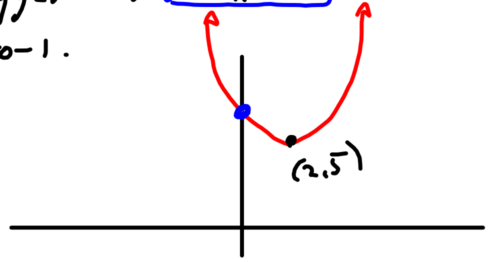
http://www.harryzaims.com/121-all/121-fall-13/tests-u-took/121-test-2-fall-13-solns-part-01-version-01.pdf

$A \Rightarrow B$ is logically equivalent to

Not $B \Rightarrow$ Not A Contrapositive

Is $3(x-2)^2+5$ 1-to-1 ?

Suggest a $\boxed{\text{maximal}}$ domain to make it 1-to-1.



2 good answers!

$\{x \mid x \geq 2\} = [2, \infty)$

oR

$\{x \mid x \leq 2\} = (-\infty, 2]$

Find $f^{-1}(x)$ if $f(x) = 3(x-2)^2+5$, $x \geq 2$

Graph both (quick sketch)

$3(y-2)^2+5 = x$

$3(y-2)^2 = x-5$

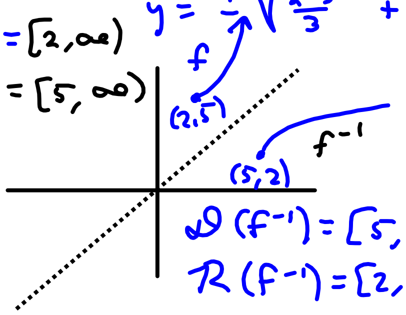
$(y-2)^2 = \frac{x-5}{3}$

$y-2 = \pm\sqrt{\frac{x-5}{3}}$

$y = \pm\sqrt{\frac{x-5}{3}} + 2$ is 2 funcs.

$\mathcal{D}(f) = [2, \infty)$

$\mathcal{R}(f) = [5, \infty)$



$\mathcal{D}(f^{-1}) = [5, \infty)$

$\mathcal{R}(f^{-1}) = [2, \infty)$

$y = \sqrt{\frac{1}{3}(x-5)} + 2$

oR

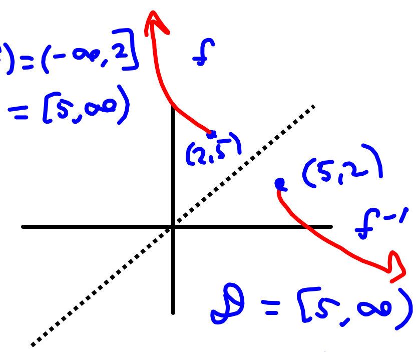
$y = -\sqrt{\frac{1}{3}(x-5)} + 2$

Find $f^{-1}(x)$ if $f(x) = 3(x-2)^2+5$, $\boxed{x \leq 2}$

Quick sketch for both.

$\mathcal{D}(f) = (-\infty, 2]$

$\mathcal{R}(f) = [5, \infty)$



$\mathcal{D} = [5, \infty)$

$\mathcal{R} = (-\infty, 2]$

$y = \sqrt{\frac{1}{3}(x-5)} + 2$

oR

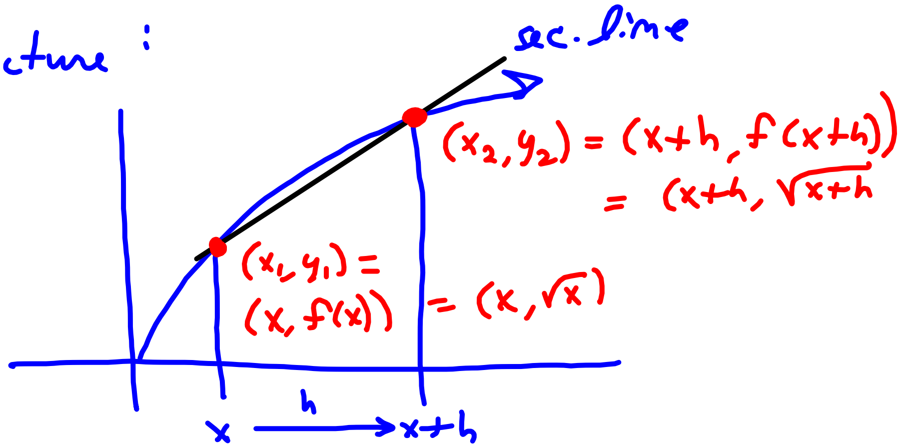
$y = -\sqrt{\frac{1}{3}(x-5)} + 2$

Oct 8-8:16 AM

Difference quotient for $f(x) = \sqrt{x}$ $\Delta x = h = x_2 - x_1$

$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

Picture:



$x_2 - x_1 = x + h - x = h$

$$\sqrt{(x+h)^2}$$
$$= |x+h|$$

Simplify it:

$$\left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right) \rightarrow \text{to rationalize numerator.}$$

$$= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{\cancel{h}}{\cancel{h}(\quad)}$$

etc.

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$\xrightarrow{h \to 0} \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Oct 8-8:44 AM

$$\left(\sqrt{x+h}\right)^2 = x+h$$

$$(a-b)(a+b) = a^2 - b^2$$