① Show that $f(x) = \frac{x-1}{x+2}$ is 1-to-1

$\beta \quad y_1 = y_2 \longrightarrow$

$$\frac{x_1-1}{x_1+2} = \frac{x_2-1}{x_2+2}$$

$$(x_1-1)(x_2+2) = (x_2-1)(x_1+2)$$

$$x_1x_2 + 2x_1 - x_2 - 2 = x_2x_1 + 2x_2 - x_1 - 2$$

$$+x_1 \qquad = \qquad +x_1$$

$$3x_1 - x_2 = 2x_2$$

$$3x_1 = 3x_2$$

$\therefore \quad x_1 = x_2$! 1-to-1 !

(2) Find f^{-1} for $f(x)=\frac{x-1}{x+2}=y$

$$\frac{y-1}{y+2}=x$$

$$y-1= x(y+2) = xy+2x$$

$$-xy+1= \qquad -xy \qquad +1$$

$$y-xy=2x+1$$

$$y(1-x)=2x+1$$

$$\boxed{y=\frac{2x+1}{1-x}} = f^{-1}(x)$$

3 Is $R = \{(1,2), (3,4), (4,3), (1,2)\}$ a function?
If so, is it 1-to-1?

Yes. Yes.

$D = \{1, 3, 4\}$ Not asked.
$R = \{2, 4, 3\}$

Oct 6-8:50 AM

4) The force of gravity on an object of mass m_1, a distance r from a planet of mass m_2 is jointly proportional to the two masses and inversely proportional to the square of the distance r, with proportionality constant G. Write an eq'n for this.

$$F = G\frac{m_1 m_2}{r^2}$$

10. (10 pts) Suppose y varies jointly with m_1 and m_2, and inversely with the square of r. Write an equation describing this situation.

Oct 6-8:52 AM

$f(x) = \sqrt{x^3 - 1}$

$g(x) = x^3$

$h(x) = x - 1$

$j(x) = \sqrt{x}$

$j(h(g))$

$f = j \circ h \circ g$

$= j(h(g(x)))$

$g(x) = x$

$h(x) = x$

$j(x) = \sqrt{x^3 - 1}$

Oct 6-7:57 AM

5. (5 pts) Simplify the difference quotient, $\frac{f(x+h)-f(x)}{h}$, for $f(x)=x^2-5x$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-5(x+h)-(x^2-5x)}{h}$$

$$= \frac{x^2+2xh+h^2-5x-5h-x^2+5x}{h} = \frac{2xh+h^2-5h}{h}$$

$$= \frac{h(2x+h-5)}{h} = 2x+h-5$$

$\lim (2x+h-5)$ is ok.

$\lim\limits_{h\to 0} =$ means nothing.

Bonus Pass to the limit, as $h \to 0$, on your answer to the above, so you can show me some calculus.

$$\xrightarrow{h \to 0} 2x-5$$

Bonus Simplify the difference quotient for $f(x)=\sqrt{x}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

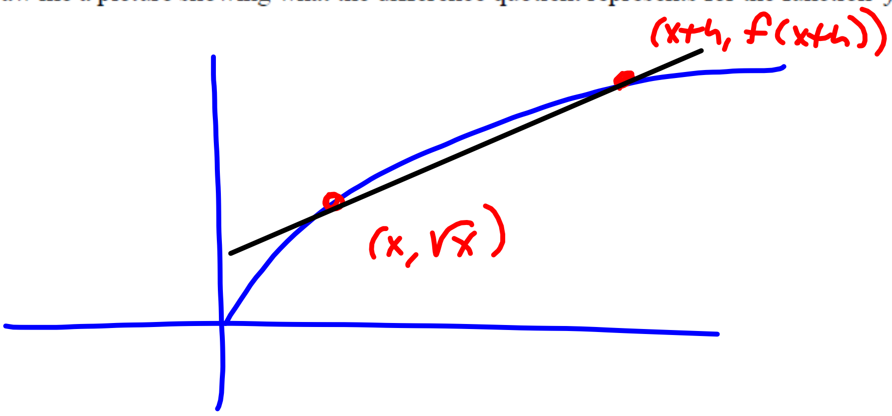
$$= \left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right) = \frac{\sqrt{x+h}^{2}-\sqrt{x}^{2}}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= \frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \frac{1}{\sqrt{x+h}+\sqrt{x}}.$$

Bonus: Pass to the limit as $h \to 0$.

$$\xrightarrow{h \to 0} \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

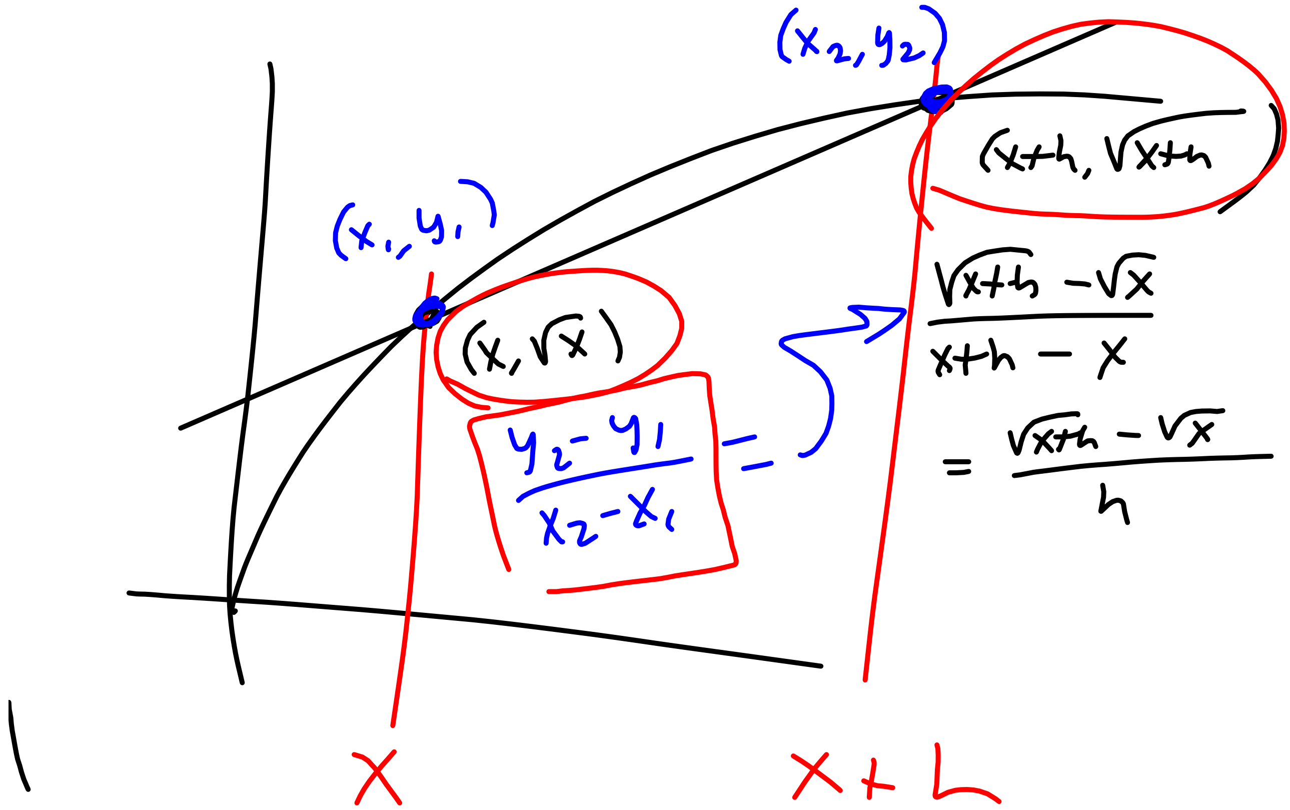
7. (5 pts) Draw me a picture showing what the difference quotient represents for the function $f(x)=\sqrt{x}$.



$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{f(x+h)-f(x)}{h} =$$

$(x+h-x=h)$

$$= \frac{\sqrt{x+h}-\sqrt{x}}{h}$$



$$\boxed{\frac{y_2 - y_1}{x_2 - x_1}} = \frac{\sqrt{x+h} - \sqrt{x}}{x+h - x} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$