§2.1 Questions #93, 54, 65

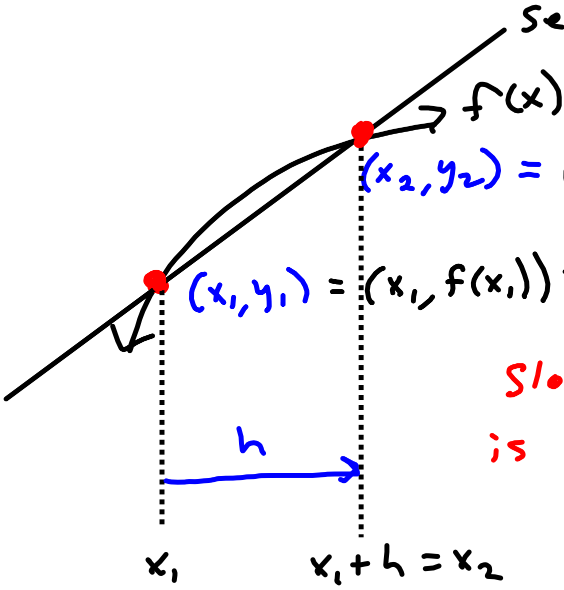
Lecture a little on §2.1

How to Study Ahead for greater efficiency.

Basic Functions

#19 on are legit questions.

#93 Difference Quotient Buildup.



$(x_2, y_2) = (x_2, f(x_2)) = (x_1+h, f(x_1+h))$

$= (x+h, f(x+h))$

$(x_1, y_1) = (x_1, f(x_1)) = (x, f(x))$

Slope of secant line

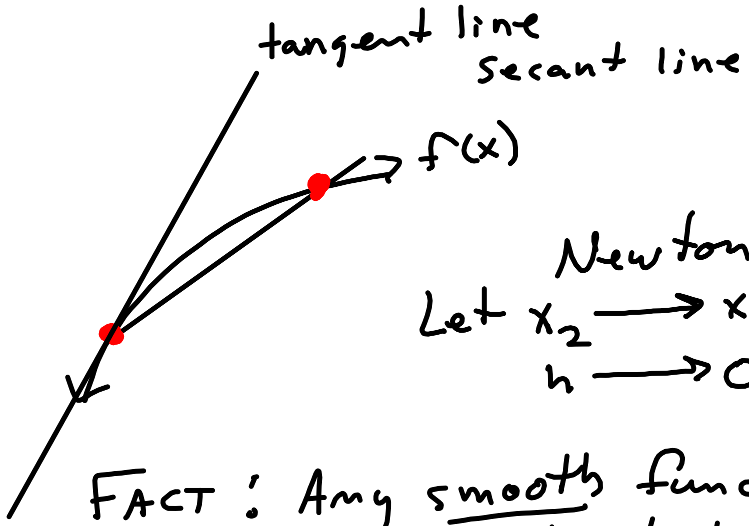
is $m_{sec} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{f(x+h) - f(x)}{h}$

= Difference Quotient

$x_2 - x_1 = x + h - x = h$

= <u>Average Slope</u> for $f(x)$ between $x = x_1$ and $x = x_2$



Newton said:
Let $x_2 \longrightarrow x_1$, i.e.
$h \longrightarrow 0$, but $h=0$ is illegal!

FACT: Any smooth function, there's always an algebra trick to "let $h=0$"

m_{tan} = EXACT SLOPE @ $x = x_1$

$\boxed{E}$ $f(x) = x^2$ $\qquad$ $f(x)+h$ No!

$f(x+h) = (x+h)^2$ $\qquad$ $(x+h)(x+h)$

$= x^2+2xh+h^2$ $\qquad$ $= x^2+2xh+h^2$

$m_{sec} = \dfrac{f(x+h)-f(x)}{h}$ $\qquad$ $(a+b)^2 = a^2+2ab+b^2$

$= \dfrac{x^2+2xh+h^2-x^2}{h}$

$= \dfrac{2xh+h^2}{h} = \dfrac{h(2x+h)}{h} = 2x+h \xrightarrow{h \to 0} 2x$

Newton says: Let $h \longrightarrow 0$

Your 1st Derivative for Calc I.

The derivative tells us exactly how steep f(x) is at any point. That's the motivation.

#93 $f(x) = 3\sqrt{x} \longrightarrow$

difference quotient $= m_{sec} = \dfrac{f(x+h) - f(x)}{h}$

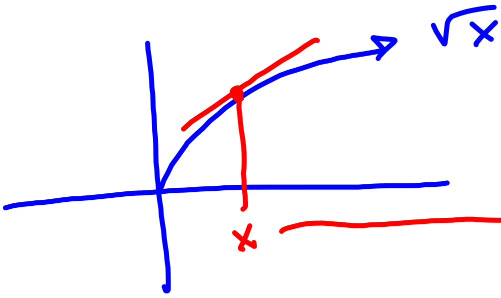
$$= \frac{3\sqrt{x+h} - 3\sqrt{x}}{h} = \frac{3\sqrt{x+h} - 3\sqrt{x}}{h} \cdot \frac{3\sqrt{x+h} + 3\sqrt{x}}{3\sqrt{x+h} + 3\sqrt{x}}$$

$$= \frac{(3\sqrt{x+h})^2 - (3\sqrt{x})^2}{h(3\sqrt{x+h} + 3\sqrt{x})} = \frac{9(x+h) - 9x}{3h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{9x + 9h - 9x}{3h(\sqrt{x+h} + \sqrt{x})} = \frac{9h}{3h(\sqrt{x+h} + \sqrt{x})} = \frac{3}{\sqrt{x+h} + \sqrt{x}}$$

Book wants that final answer, so Newton can say "Let $h \to 0$". This gives $\dfrac{3}{\sqrt{x} + \sqrt{x}} = \dfrac{3}{2\sqrt{x}}$

$$(3\sqrt{x+h})^2 = (AB)^2 = A^2B^2 = 3^2(\sqrt{x+h})^2$$
$$= 9(x+h)$$



KNOW IT!

$(A-B)(A+B) = A^2 - B^2$

$(A-B)^2 = A^2 - 2AB + B^2$

$(A+B)^2 = A^2 + 2AB + B^2$

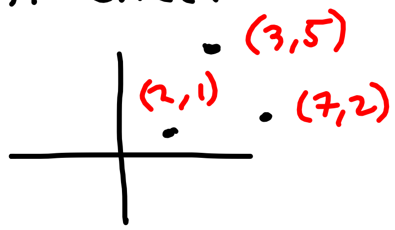
$\boxed{\text{Function } x \text{ can't repeat}}$

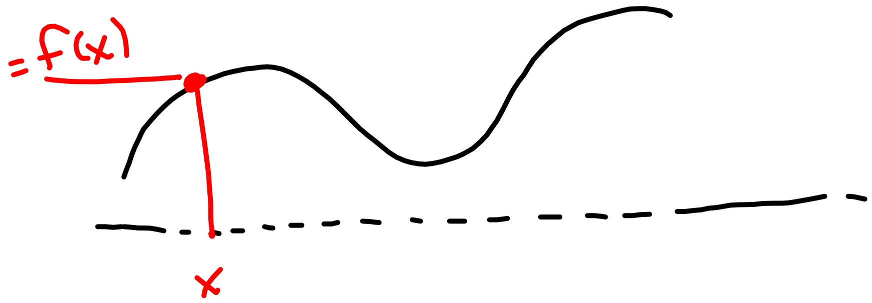
Relation x can "

Vertical Line can only hit it once.

$f = \{(2,1), (3,5), (7,2)\}$

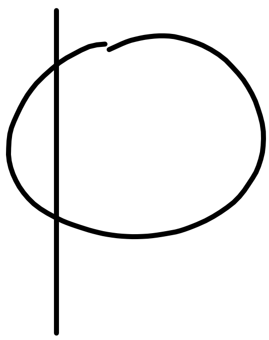
$f(2) =$ "f of '2'" is 1





All functions are relations, but not all relations are functions

$x^2+y^2=9$ is a circle of radius 3, centered @ $(0,0)$. It's a relation, but not a function



The points $(0,3)$ and $(0,-3)$ are on it / in it

To see if a formula describes y as a function of x, solve for y & see if there's a single-valued expression on the RHS.

$$x^2+y^2=9$$
$$y^2=9-x^2$$
$$\sqrt{y^2}=\sqrt{9-x^2}$$
$$|y|=\sqrt{9-x^2}$$
$$y=\pm\sqrt{9-x^2}$$ is <u>not</u> single-valued.

The "$\pm$" screws it up!

$$x^2=y^2$$
$$\sqrt{x^2}=\sqrt{y^2}$$
$$|x|=|y|$$

$\boxed{y^{2n} \text{ Not}}$ $\quad y^{\text{even}}$ Not

y^{2n+1} Probably $\quad y^{\text{odd}}$ Prob'ly yes. $\quad y=\pm x$

Sep 19-8:40 AM

Domain $= \mathcal{D}(f) = \{x \mid x \text{ is "legal"}\}$

Range $= \mathcal{R}(f) = \{y \mid y = f(x) \underbrace{\text{for a legal } x}_{\text{for } x \in \mathcal{D}(f)}\}$

$G = \{(1,2), (3,5), (7,-6)\}$

$\mathcal{D} = \{1,3,7\}$ Yes, function.

$\mathcal{R} = \{2,5,-6\}$

Domain Concerns

1. $\frac{\text{Stuff}}{0}$ Bad
2. $\sqrt{\text{negative}}$ Bad

Everything else is all good.

$\mathcal{D} = (-\infty, \infty)$, with only the two exceptions, above. More in sequel.

Sep 19-8:45 AM

Prep for class

§2.4 Top of Page

Sum, Difference, Product and Quotient funcs.

$(f+g)(x) = f(x)+g(x)$ Sum

$(f-g)(x) = f(x)-g(x)$ Difference

$(f\cdot g)(x) = \boxed{(fg)(x)}$ Product

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Start new page.

1 Composition of Functions

$(f\circ g)(x) = \boxed{f(g(x))}$ Composition

Sep 19-8:52 AM