Systems of Linear Inequalities

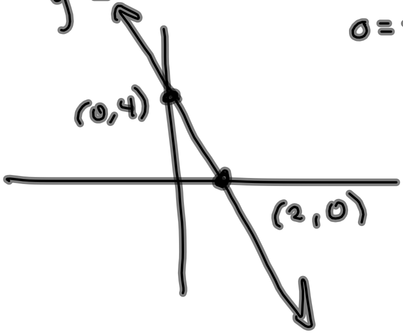
Scratch out the Bad Stuff!

FACT: A line divides the plane into 2 half-planes

Wait! You need the 10-minute spiel on graphing lines!

Stephen

$y = -2x + 4$



x-int:

$y = 0$

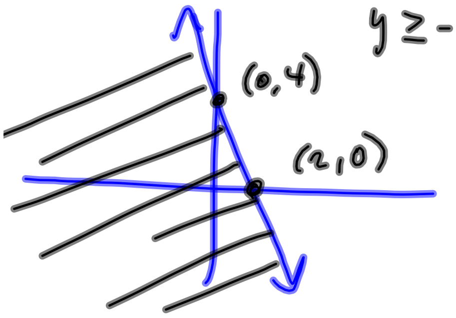
$0 = -2x + 4 = 0$

$-2x = -4$

$x = \frac{-4}{-2} = +2$

Test Value Method

$y \geq -2x + 4$



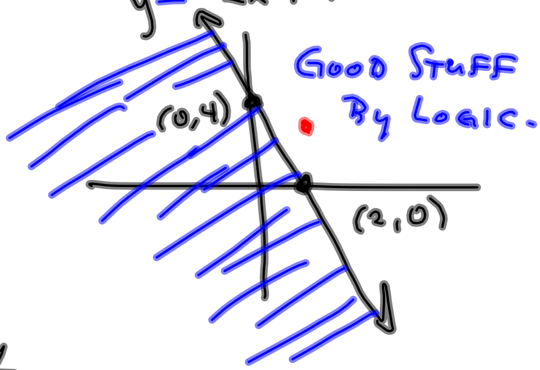
Test: (0,0)

$0 \geq -2(0) + 4$?

$0 \geq 4$?

No! (0,0) Bad.

$y \geq -2x + 4$



x	y
0	4
2	0

$y = -2(0) + 4 = 4$

$0 = -2x + 4 = 0$

$-2x = -4$

$x = \frac{-4}{-2} = 2$

Lines: Intercepts are the biggie, followed by slope (when the intercept is the origin).

otherwise, lines have 2 intercepts.

$3x - 2y = 6$

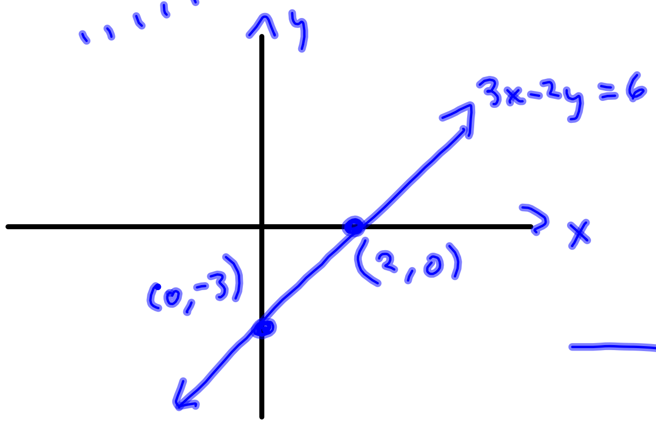
x	y
0	-3
2	0

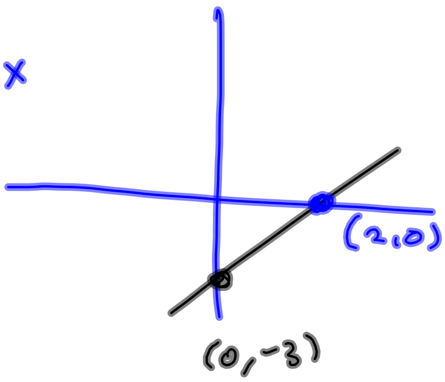
$-2y = 6$

$y = \frac{6}{-2} = -3$

$3x = \cancel{6}$

$x = \frac{6}{3} = 2$

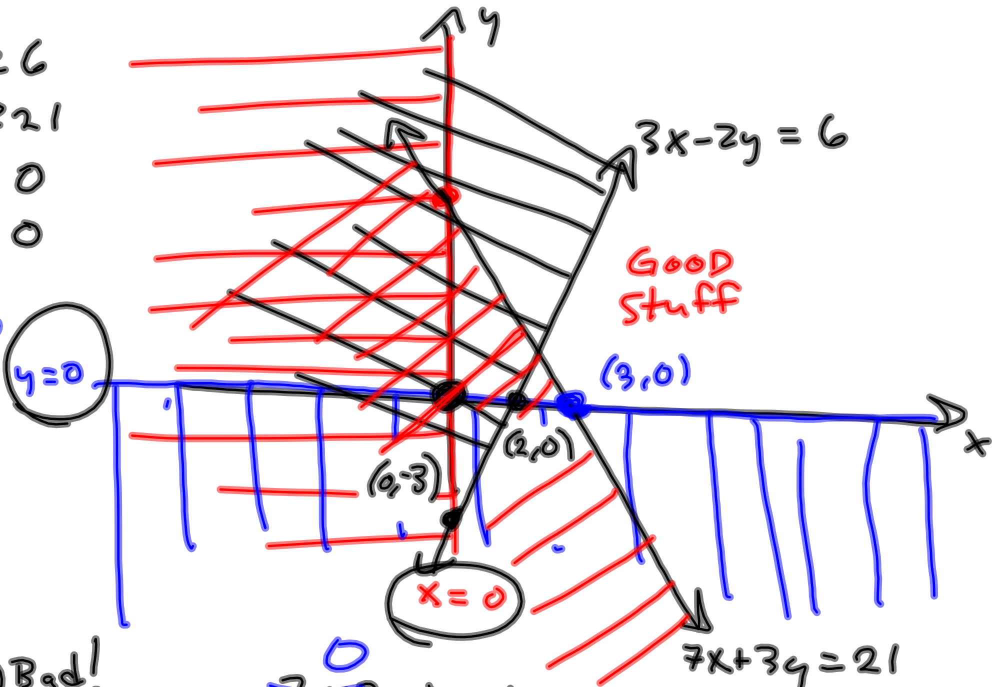




$3x-2y \geq 6$
$7x+3y \geq 21$
$x \geq 0$
$y \geq 0$

$x=0$
$x \geq 0$

$y=0$
$y \geq 0$



$3x-2y \geq 6$

x	y
0	-3
2	0

(0,0): $0 \geq 6$?
No, (0,0) Bad!

$\geq$ ————

$>$ - - - - - - - -

No big deal to me.

$\cancel{7x+3}y \geq 21$ (0)

x	y
0	7
3	0

(0,0)
$0 \geq 21$?
No (0,0) Bad.

$x = 5$ "x is 5"

$x \in \{5\}$ "x is in the set containing the number 5."

solution set $\{5\}$

3 types of equation:

1. Conditional – Most of the ones we do.
2. Identity – True $\forall\, x$
3. Inconsistent – No solution

$\forall$: For each, For all, For every.

$\exists$: There is, there exists.

$\ni$: Such that, so that.

1. Later
2. Identity $\frac{1}{x-1} - \frac{1}{2x-2} = \frac{1}{2x-2}$ LCD: $2(x-1)$

$2x-2 = 2(x-1)$ $2x-2 = 2(x-1)$

$$\frac{1}{x-1}\cdot\frac{2}{2} - \frac{1}{2(x-1)} = \frac{1}{2(x-1)}$$

$$\boxed{\begin{array}{l}\text{IDENTITY}\\ x \in \{x \mid x \neq 1\}\end{array}}$$

$$\frac{2-1}{LCD} = \frac{1}{LCD}$$

$$2-1 = 1$$

$$1 = 1$$

Tautology – True, regardless of value of variable(s)

Vacuous truth.

$$x \in \{z \mid z \neq 1\} = \{\xi \mid \xi \neq 1\} = (-\infty, 1) \cup (1, \infty)$$

Representative of the set.

Conditions on any member or representative.

Set-Builder Notation.

$\frac{1}{x-1}$ $\nexists$ when $x = 1$

$\frac{1}{1-1} = \frac{1}{0}$ is not defined as a real number.

③ Inconsistent

$$\frac{3}{x-1} - \frac{1}{2x-2} = \frac{1}{2x-2}$$

LCD = 2(x-1)

$$\frac{3}{x-1}\cdot\frac{2}{2} - \frac{1}{2(x-1)} = \frac{1}{2(x-1)}$$

Rationalization of denominator isn't what's happening, here.

$$\frac{6-1}{LCD} = \frac{1}{LCD}$$

$$6-1 = 1$$

$$5=1$$

FALSE

No Solution!

Reductio Ad Absurdum

Reasoning from a false premise leads to something ridiculous!

Every move we made to solve this equation was based on the (false) assumption that there WAS a solution.

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

we've rationalized the denominator

irrational

rational

$$\frac{2}{\sqrt{3}+5} = \frac{2}{(\sqrt{3}+5)}\left(\frac{\sqrt{3}-5}{(\sqrt{3}-5)}\right) = \frac{2\sqrt{3}-10}{-22}$$

$$(a+b)(a-b) = a^2 - b^2$$

$\sqrt{3}^2 - 5^2$

$3-25$

$=-22$

Not current topic.

Completing the Square

$x^2 - 8x - 19 = 0$

Completing Square to solve

$x^2 - 8x = 19$

$\frac{8}{2} = 4 \rightsquigarrow 4^2$

$x^2 - 8x + 4^2 = 19 + 16$

$\sqrt{(x-4)^2} = \sqrt{35}$

$|x-4| = \sqrt{35}$

$x - 4 = \pm\sqrt{35}$

$x = 4 \pm \sqrt{35}$

An expression to be manipulated not an equation to be solved.

$$y = \underline{x^2 - 8x - 19} = x^2 - 8x + 4^2 - 16 - 19$$

Completing the square for graphing.

$$\frac{8}{2} = 4 \longrightarrow 4^2 = 16$$

$$a(x-h)^2 + k$$

$(h, k) =$ vertex

$$= (x-4)^2 - 35$$

$(4, -35)$