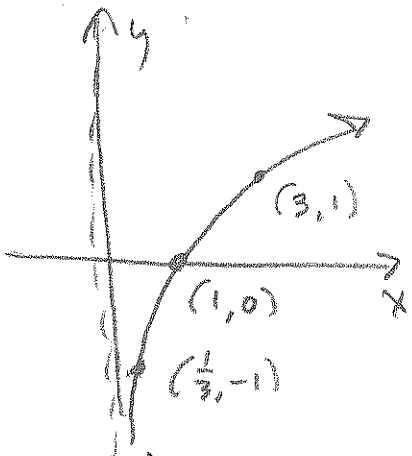
MAKEUP

MAT 121
70 Points

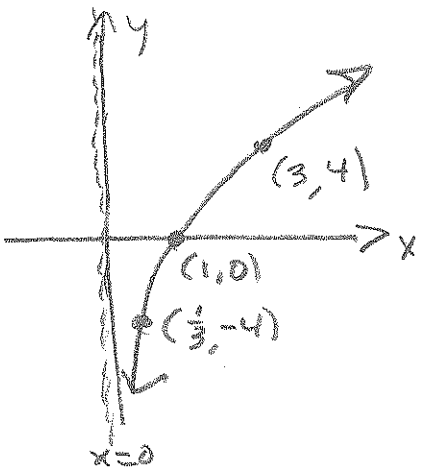
Test 4 - Fall, 2014
Covers Chapter 4

Name KEY

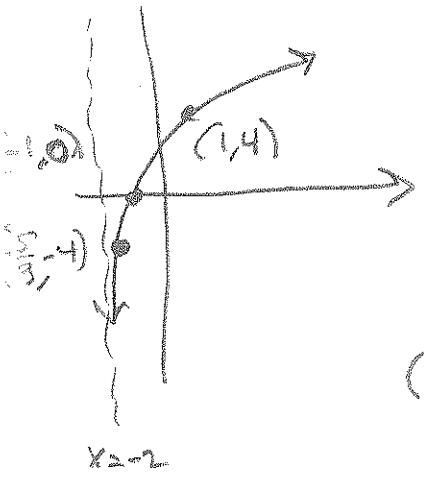
1. (15 pts) Starting with $f(x)=\log_3(x)$, sketch the graph of $g(x)=4\cdot\log_3(x+2)-3$ in 4 steps (counting $f(x)=\log_3(x)$ as the first step). Use $x=\frac{1}{3}$, $x=1$, and $x=3$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Finding the x- and y-intercepts is a separate problem, so don't worry about them, on this page.



$f(x)=\log_3(x)$



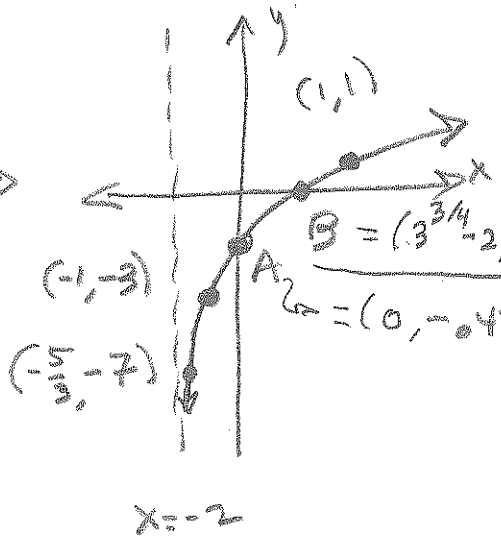
$4f(x)=4\log_3(x)$



$4\log_3(x+2)$

$=4f(x+2)$

$\frac{1}{3}-2=\frac{1-6}{3}=-\frac{5}{3}$



A: $g(0)=4\log_3(2)-3$

$A=(0,4\log_3(2)-3)$

$\approx(0,-.4763)$

B: $(3^{3/4}-2,0)\approx(.2795,0)$

$A_2=(0,-.4763)$

B: $g(x)=0$

$4\log_3(x+2)-3=0$

$4\log_3(x+2)=3$

$\log_3(x+2)=\frac{3}{4}$

$x+2=3^{3/4}$

$x=3^{3/4}-2\approx .2795$

2. Let $f(x)=\sqrt{2x+4}$ and $g(x)=\dfrac{x-2}{x-7}$.

a. (5 pts) What is the domain of f?

Need $2x+4 \geq 0$

$2x \geq -4$

$D = \{x \mid x \geq -2\} = [-2, \infty)$

b. (5 pts) What is the domain of g?

Need $x-7 \neq 0$

$D = \{x \mid x \neq 7\} = (-\infty, 7) \cup (7, \infty)$

c. (5 pts) Write the function $\dfrac{f}{g}$. Do not simplify.

$$\frac{\sqrt{2x+4}}{\frac{x-2}{x-7}}$$

d. (5 pts) Write the function $f \circ g$. Do not simplify.

$$\sqrt{2\left(\frac{x-2}{x-7}\right)+4}$$

e. (10 pts) What is the domain of $\dfrac{f}{g}$?

Need $x \in D(f)$ and $x \in D(g)$ and $g(x) \neq 0$:

$D = \{x \mid x \geq -2 \text{ and } x \neq 7 \text{ and } \frac{x-2}{x-7} \neq 0\}$

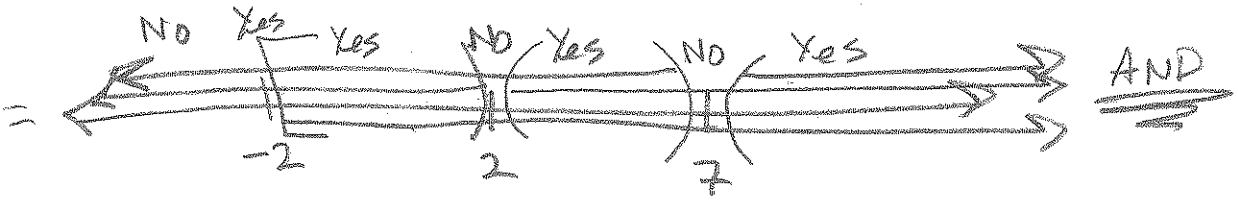
$= \{x \mid x \geq -2 \text{ and } x \neq 7 \text{ and } x \neq 2\}$

SCRATCH

$\frac{x-2}{x-7} \neq 0$

$x-2 \neq 0$

$x \neq 2$



$= [-2, 2) \cup (2, 7) \cup (7, \infty)$

3. (5 pts) Let $g(x)=4\cdot\log_3(x+2)-3$. Find the x- and y-intercepts for this function, rounded to 4 decimal places. For 5 **bonus** points, label these intercepts on your final graph on page 1.

See Pg 1, also

$4\log_3(x+2)-3=0$

$4\log_3(x+2)=3$

$\log_3(x+2)=\frac{3}{4}$

$x+2=3^{\frac{3}{4}}$

$x=3^{\frac{3}{4}}-2\approx .2795070057$

$(.2795, 0)$ is x-int

$g(0)=4\log_3(2)-3$

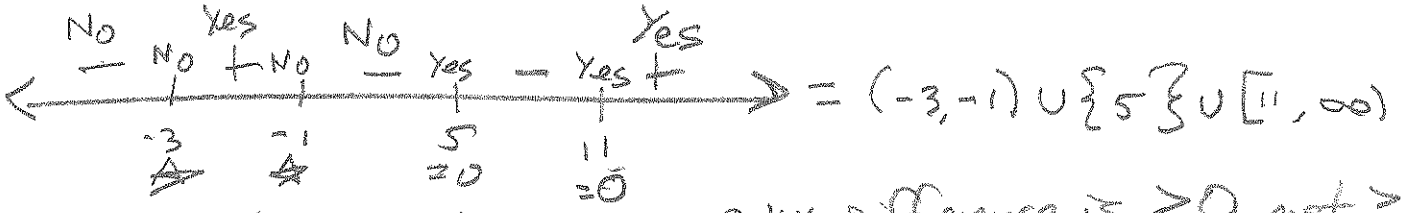
$\approx -.4762809857$

$(0, -.4763)$ is y-int

4. Find the domain:

a. (5 pts) $\sqrt{\dfrac{(x-5)^2(x-11)^3}{(x+1)^5(x+3)}}$

Need $\dfrac{(x-5)^2(x-11)^3}{(x+1)^5(x+3)}\geq 0$ and $x\neq -1$ and $x\neq -3$



$=(-3,-1)\cup\{5\}\cup[11,\infty)$

ONLY Difference is >0, not ≥ 0.

b. (5 pts) $\log_3\left(\dfrac{(x-5)^2(x-11)^3}{(x+1)^5(x+3)}\right)$ (Reinterpret previous work and write the answer.

Same, but $x=5$ & $x=11$ are out. $(-3,-1)\cup(11,\infty)$

5. (5 pts) Solve $\log(x-7)+\log(x+2)=1$

$\log((x-7)(x+2))=1$

$(x-7)(x+2)=10^1$

$x^2-5x-14=10$

$x^2-5x-24=0$

$(x-8)(x+3)=0$

$x\in\{8\}$

$x\in\{-3,8\}$, but $x=-3\neq 0$!

Check $x=8$: $\log(1)+\log(10)=1$ ✓

6. (5 pts) Solve $3^{x^2-8}\cdot 3^{-3x}=9$

$3^{x^2-3x-8}=3^2$

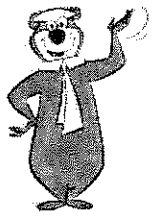
$x^2-3x-8=2$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x\in\{-2,5\}$

Solve any two (2) Bonus problems for up to 10 points. I'll grade the first two I come to.



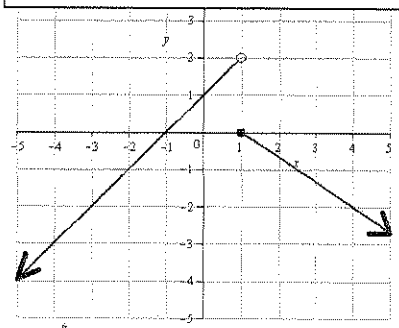
1. BONUS (5 pts) Solve the absolute value inequality $|2x-7|<8$

2. **BONUS** (5 pts) Find the inverse function for $f(x)=\sqrt{2x+8}+11$. Then state the domain and range for both f and f^{-1}.

3. BONUS (5 pts) Re-write the function $g(x)=3x^2+6x-19$ in the form $g(x)=a(x-h)^2+k$. State the vertex of this parabola.

4. BONUS (5 pts) Write the formula for the piecewise-defined function shown on the right.

This is the picture for **Bonus #4**

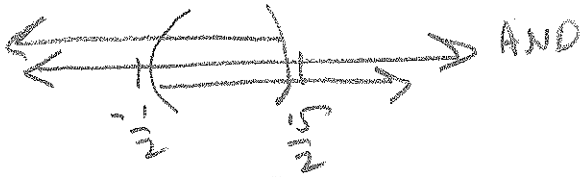


B1 $|2x-7|<8$

$2x-7<8$ AND $2x-7>-8$

$2x<15$ AND $2x>-1$

$\{x \mid x<\frac{15}{2}$ AND $x>-\frac{1}{2}\}$



$=\left(-\frac{1}{2}, \frac{15}{2}\right)$

B2 $\sqrt{2y+8}+11=x$

$\sqrt{2y+8}=x-11$

$2y+8=(x-11)^2$

$2y=(x-11)^2-8$

$y=\frac{1}{2}(x-11)^2-4=f^{-1}(x)$

$\sqrt{2y+8}+11=\sqrt{2(y+4)}+11$

$(-4,11)$

$D(f)=[-4,\infty)=R(f^{-1})$

$R(f)=[11,\infty)=D(f^{-1})$

B3 $3x^2+6x-19$

$=3(x^2+2x+1^2)-19-3(1)^2$

$=3(x+1)^2-22$

$(h,k)=(-1,-22)$

Alt: $-\frac{b}{2a}=\frac{-6}{2(3)}=-1=h$

$f(-\frac{b}{2a})=3(-1)^2+6(-1)-19$

$=3-6-19=-22=k$

B4 l_1: $(1,2),(0,1)$

$y=\frac{1-2}{0-1}(x-0)+1=x+1$

l_2: $(1,0),(4,-2)$

$y=\frac{-2-0}{4-1}(x-1)+0=-\frac{1}{2}(x-1)=-\frac{1}{2}x+\frac{1}{2}$

$$f(x)=\begin{cases} x+1 & \text{if } x<1 \\ -\frac{1}{2}x+\frac{1}{2} & \text{if } x\geq 1 \end{cases}$$

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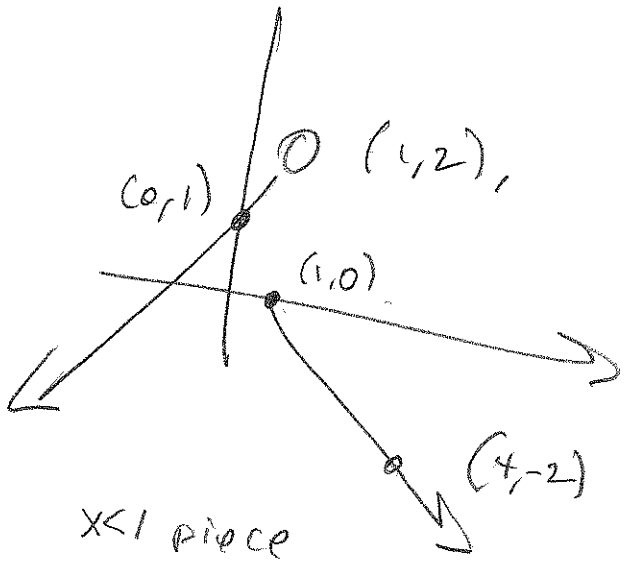
TEST 4 MAKEUP – ERRATA



$x<1$ piece

$m = \frac{2-1}{1-0} = 1$

$y = 1(x-0)+1$

$y = x+1$

$x \geq 1$ piece

$m = \frac{-2-0}{4-1} = -\frac{2}{3}$

$y = -\frac{2}{3}(x-1)+0$

$y = -\frac{2}{3}x + \frac{2}{3}$

$$f(x) = \begin{cases} x+1 & \text{if } x<1 \\ -\frac{2}{3}x+\frac{2}{3} & \text{if } x \geq 1 \end{cases}$$