MAT 121-G11 Test 4 Take-Home – Fall, 2014 Name_____

60 pts

Due Monday, November 17th. I will have *zero* patience for stapling during class time. I will have zero tolerance for late work.

Use separate paper to do the work on this take-home test. Make sure your pencil work is *dark*. It's a struggle for me to read faint print, and I'm done with it costing *me* time and stress. If I can't read it, easily, that's a zero, and I'm moving on.

1. (5 pts) Starting with $f(x)=4^x$, sketch the graph of $g(x)=2\cdot 4^{-3x-3}-9$ in 5 steps (counting $f(x)=4^x$ as the first step). Use $x=-1$, $x=0$, and $x=1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Include the x- and y-intercepts. to 4 places
2. (5 pts) Starting with $h(x)=\log_4(x)$, sketch the graph of $w(x)=-2\log_4(x+9)-7$ in 4 steps (counting $h(x)=\log_4(x)$ as the first step.) Use $x=\frac{1}{4}$, $x=1$, and $x=4$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformations to $w(x)$. Include x- and y-intercepts.
3. Let $f(x)=\sqrt{2x+4}$ and $g(x)=\dfrac{x-2}{x-7}$. to 4 places.

   a. (5 pts) What is the domain of f?

   b. (5 pts) What is the domain of g?

   c. (5 pts) Write the function $\frac{g}{f}$. Do not simplify.

   d. (5 pts) Write the function $g\circ f$. Do not simplify.

   e. (5 pts) What is the domain of $\frac{g}{f}$?

   f. (5 pts) What is the domain of $g\circ f$?

4. Find the domain:

   a. (5 pts) $\sqrt{\dfrac{(x-3)(x+4)^2}{(x-8)^4(x+6)}}$

   b. (5 pts) $\log_3\left(\dfrac{(x-3)(x+4)^2}{(x-8)^4(x+6)}\right)$

5. (5 pts) Re-write $\ln\left(\dfrac{\sqrt[5]{x^2y}}{t^{3/4}}\right)$ as a sum or difference of multiples of (simpler) logarithms.
6. (5 pts) Re-write $3\log_4(x^2)-\log_4(x^3)+2\log_4(\sqrt[4]{x})$ as a single logarithm.
7. (5 pts) The half-life of a radioactive isotope is 100 years. How old is a sample of that isotope if 93% of it has decayed into other by-products?
8. (5 pts) How much should I put into an account earning 7% APR, compounded weekly, if I want to have $10,000 in the account in 5 years?

1. **BONUS** (5 pts) Find the inverse function for $f(x)=\sqrt{2x-6}+1$. Then state the domain and range for both f and f^{-1}.

2. **BONUS** (5 pts) Re-write the function $g(x)=5x^2+10x-19$ in the form $g(x)=a(x-h)^2+k$. State the vertex of this parabola.

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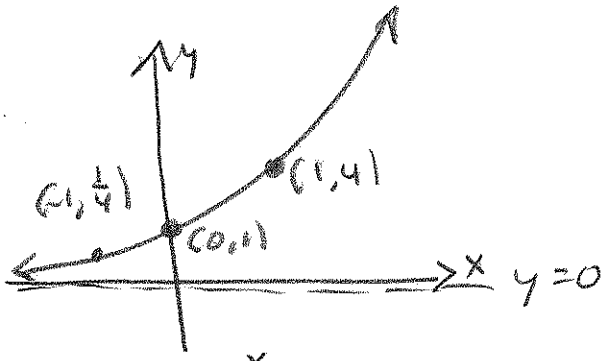
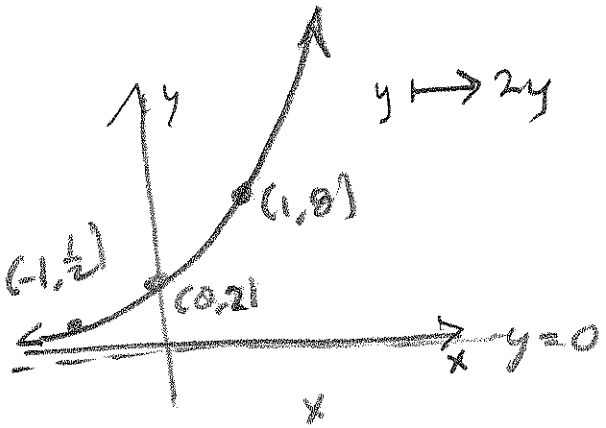
① $g(x) = 2 \cdot 4^{3x-3} - 9$

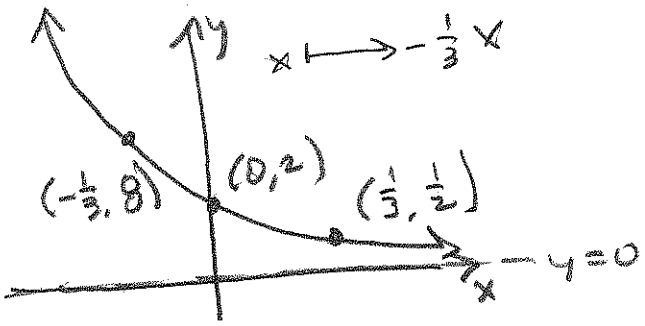
$-3x-3 = -3(x+1)$



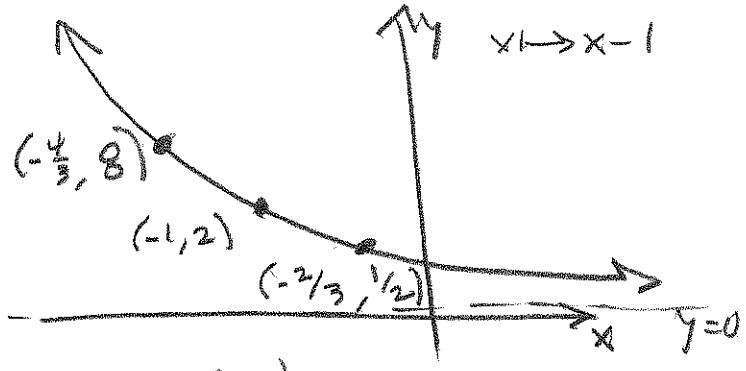
$f(x) = 4^x$



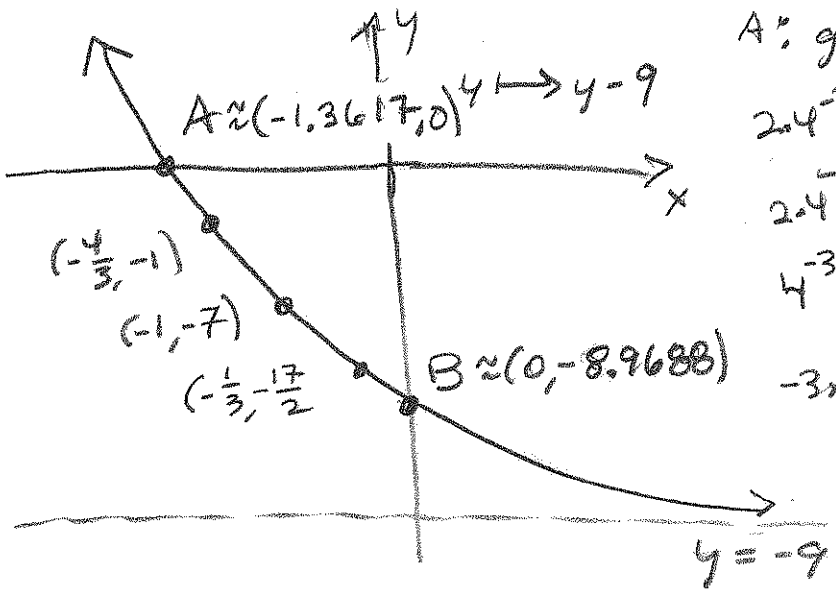
$2f(x) = 2 \cdot 4^x$



$2 \cdot 4^{-3x} = 2 \cdot f(-3x)$



$2 \cdot 4^{-3(x+1)} = 2f(-3(x+1))$



$2 \cdot 4^{-3(x+1)} - 9 = g(x)$

$= 2f(-3(x+1)) - 9$

A: $g(x) = 0$

$2 \cdot 4^{-3x-3} - 9 = 0$

$2 \cdot 4^{-3x-3} = 9$

$4^{-3x-9} = \frac{9}{2}$

$-3x-9 = \log_4(9/2)$

$-3x = \log_4(9/2) + 3$

$x = -\frac{1}{3}\log_4(9/2) - 1$

B: $g(0)$

$= 2 \cdot 4^{-3} - 9$

$= \frac{2}{64} - 9$

$= \frac{1}{32} - \frac{288}{32} = -\frac{287}{32}$

$B = (0, -\frac{287}{32})$

$\approx (0, -8.9688)$

$A = (-\frac{1}{3}\log_4(9/2) - 1, 0)$

$\approx (-1.3617, 0) \approx A$

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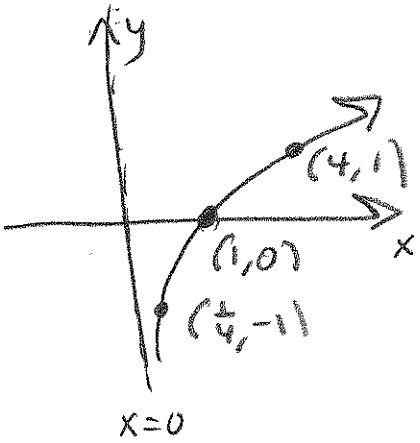
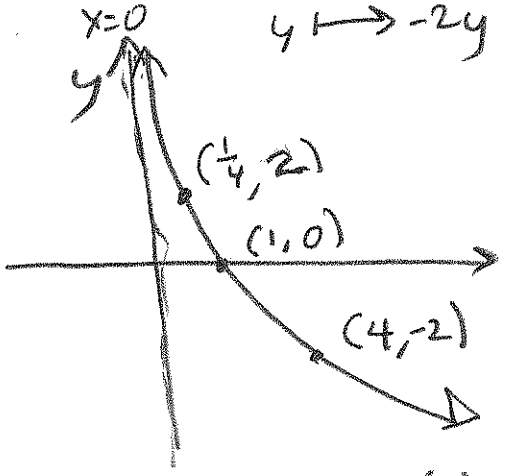
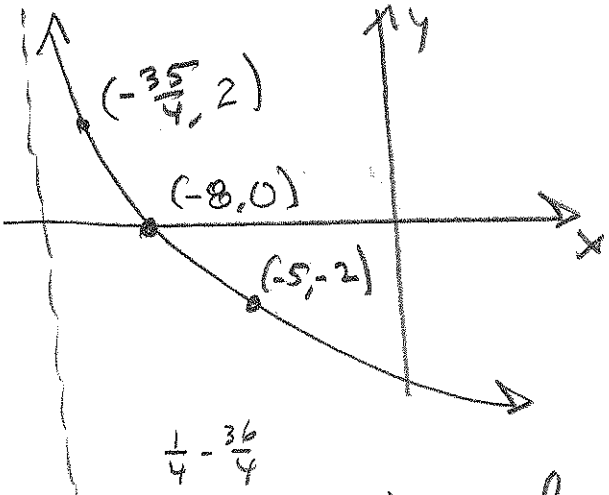
(2) $w(x) = -2\log_4(x+9) - 7$



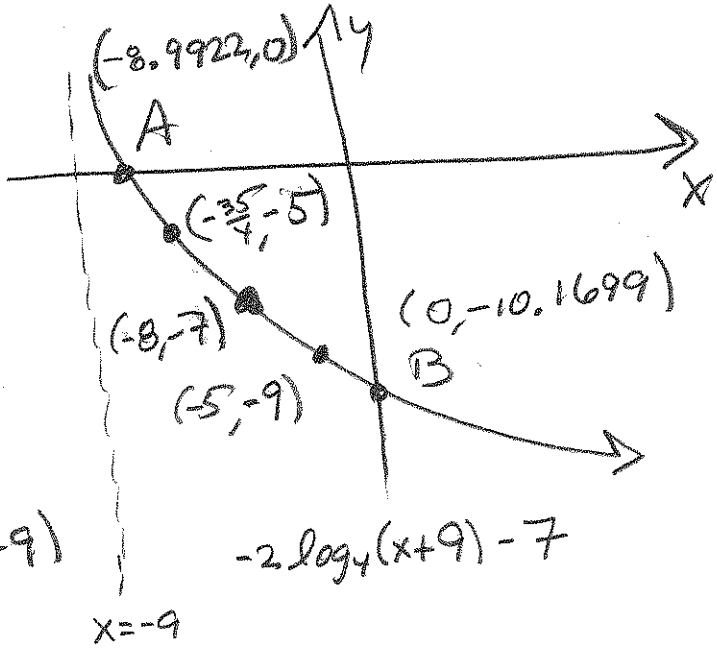
$h(x) = \log_4(x)$



$-2h(x) = -2\log_4(x)$



$-2h(x+9) = -2\log_4(x+9)$



$-2\log_4(x+9) - 7$

A: $w(x) = 0$

$-2\log_4(x+9) - 7 = 0$

$-2\log_4(x+9) = 7$

$\log_4(x+9) = -7/2$

$x+9 = 4^{-7/2} = 2^{-7} = \frac{1}{128}$

$x = \frac{1}{128} - 9 = \frac{1-1152}{128} = -\frac{1151}{128}$

$\rightsquigarrow \left(-\frac{1151}{128}, 0\right) \approx (-8.9922, 0) \approx A$

B: $w(0) = -2\log_4(9) - 7 \approx -10.1699$

$\rightsquigarrow (0, -2\log_4(9) - 7) = B$

$\rightsquigarrow (0, -10.1699) \approx B$

(3) $f(x)=\sqrt{2x+4}$, $g(x)=\dfrac{x-2}{x-7}$

(a) $D(f)=\{x \mid 2x+4\geq 0\}=\{x \mid x\geq -2\}=\boxed{[-2,\infty)}$

(b) $D(g)=\{x \mid x-7\neq 0\}=\{x \mid x\neq 7\}=\boxed{(-\infty,7)\cup(7,\infty)}$

(c) $\dfrac{g}{f}=\boxed{\dfrac{\frac{x-2}{x-7}}{\sqrt{2x+4}}}$

(d) $g\circ f=\boxed{\dfrac{\sqrt{2x+4}-2}{\sqrt{2x+4}-7}}$

(3e)
$f(x)\neq 0$
$\sqrt{2x+4}\neq 0$
$2x+4\neq 0$
$2x\neq -4$
$x\neq -2$

(3a)
$2x+4\geq 0$
$2x\geq -4$
$x\geq -2$

(e) $D\left(\frac{g}{f}\right)=\{x \mid x\in D(g) \text{ and } x\in D(f) \text{ and } f(x)\neq 0\}$

$=\{x \mid x\neq 7 \text{ and } x\geq -2 \text{ and } x\neq -2\}$

$=\{x \mid x\neq 7 \text{ and } x>-2\}=\boxed{(-2,7)\cup(7,\infty)}$

(f) $D(g\circ f)=\{x \mid x\in D(f) \text{ and } f(x)\in D(g)\}$

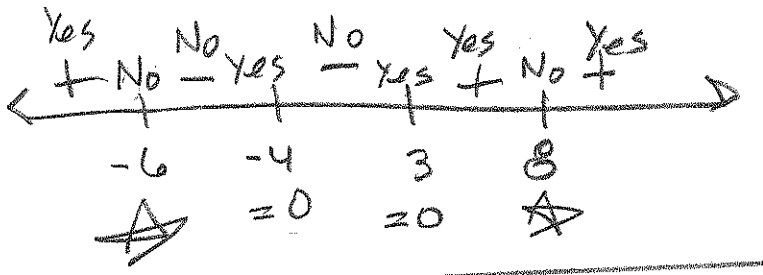
$=\{x \mid x\geq -2 \text{ and } \sqrt{2x+4}\neq 7\}$

$=\{x \mid x\geq -2 \text{ and } x\neq \frac{45}{2}\}$

$=\boxed{[-2,\frac{45}{2})\cup(\frac{45}{2},\infty)}$

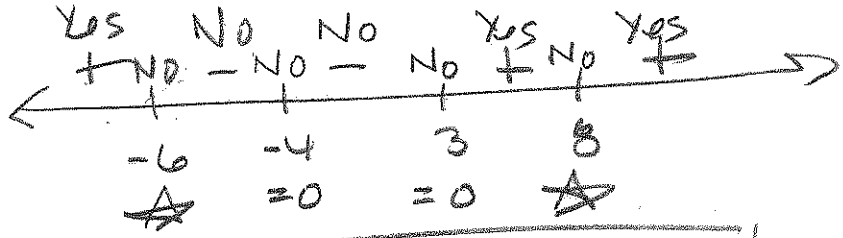
$\sqrt{2x+4}\neq 7$
$2x+4\neq 49$
$2x\neq 45$
$x\neq \frac{45}{2}=22.5$

(4a) Domain of $\sqrt{\dfrac{(x-3)(x+4)^2}{(x-8)^4(x+6)}}$ Need $\dfrac{(x-3)(x+4)^2}{(x-8)^4(x+6)} \geq 0$



$D = (-\infty, -6) \cup \{-4\} \cup [3, 8) \cup (8, \infty)$

(4b) Domain of $\log_3\left(\dfrac{(x-3)(x+4)^2}{(x-8)^4(x+6)}\right)$ NEED $\dfrac{(x-3)(x+4)^2}{(x-8)^4(x+6)} > 0$



$D = (-\infty, -6) \cup (3, 8) \cup (8, \infty)$

(5) $\ln\left(\frac{\sqrt[5]{x^3y}}{t^{3/4}}\right) = \ln\left(\frac{x^{3/5}y^{1/5}}{t^{3/4}}\right)$

$= \ln(x^{3/5}) + \ln(y^{1/5}) - \ln(t^{3/4})$

$= \frac{3}{5}\ln(x) + \frac{1}{5}\ln(y) - \frac{3}{4}\ln(t)$

(6) $3\log_4(x^2) - \log_4(x^3) + 2\log_4(\sqrt[4]{x})$

$= \log_4((x^2)^3) - \log_4(x^3) + \log_4\left((x^{\frac{1}{4}})^2\right)$

$= \log_4(x^6) - \log_4(x^3) + \log_4(x^{\frac{1}{2}})$

$= \log_4\left(\frac{x^6 \cdot x^{\frac{1}{2}}}{x^3}\right) = \log_4\left(x^{6+\frac{1}{2}-3}\right) = \boxed{\log_4(x^{7/2}) = \log_4(\sqrt[2]{x^7})}$

$\frac{1}{2} = .5$ 100
20
25 30
.125
.0625

(7) $\frac{1}{2}$-life is 100 yrs

$P_0 e^{100K} = \frac{1}{2}P_0$

$e^{100K} = \frac{1}{2}$

$100K = \ln(1/2)$

$K = \frac{\ln(1/2)}{100} = -\frac{\ln(2)}{100}$

$\approx .006931471 8 \approx K$

93% is gone
7% remains.

$P_0 e^{Kt} = .07P_0$

$e^{Kt} = .07$

$Kt = \ln(.07)$

$t = \frac{\ln(.07)}{K} = \frac{\ln(.07)}{-\frac{\ln(2)}{100}}$

≈ 383.65

$\approx$ 384 yrs old

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(8) $P(t) = P_0\left(1+\frac{r}{m}\right)^{mt}$ = Future value. $r=.07, m=52$

WANT $P(5) = P_0\left(1+\frac{.07}{52}\right)^{(52)(5)} = 10000$ \$10,000 in 5 years.

$P_0\left(1+\frac{.07}{52}\right)^{260} = 10,000$

$P_0 = \frac{10,000}{\left(1+\frac{.07}{52}\right)^{260}}$ OR $10,000\left(1+\frac{.07}{52}\right)^{-260}$

$\approx$ \$7048.54

Check: $7048.54\left(1+\frac{.07}{52}\right)^{(52)(5)} \approx \$10,000$ ✓

(B1) $f(x) = \sqrt{2x-6}+1$

$\mathcal{D}(f) = [3,\infty) = \mathcal{R}(f^{-1})$

$\mathcal{R}(f) = [1,\infty) = \mathcal{D}(f^{-1})$

$\sqrt{2y-6}+1 = x$

$\sqrt{2y-6} = x-1$

$2y-6 = (x-1)^2$

$2y = (x-1)^2+6$

$y = \frac{1}{2}(x-1)^2+3 = f^{-1}(x)$ OR $\frac{1}{2}(x^2-2x+1)+3$

$= \frac{1}{2}x^2-x+\frac{1}{2}+3 =$

$= \frac{1}{2}x^2-x+\frac{7}{2}$

$(h,k) = (1,3)$ we use right half of its graph as inverse for $f(x)$

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B2 $g(x) = 5x^2 + 10x - 19$

$= 5(x^2 + 2x + 1^2) - 19 - 5(1)^2$

$= \boxed{5(x+1)^2 - 24}$

$(h, k) = (-1, -24)$

Alternate:

$a = 5,\ b = 10,\ c = -19$

$-\frac{b}{2a} = -\frac{10}{2(5)} = -1 = h$

$f(-\frac{b}{2a}) = f(-1) = 5(-1)^2 + 10(-1) - 19$

$= 5 - 10 - 19$

$= -5 - 19$

$= -24 = k$

$f(x) = a(x-h)^2 + k$

$= 5(x-(-1))^2 + (-24)$

$= 5(x+1)^2 - 24$

Check/Cheat for completing the square