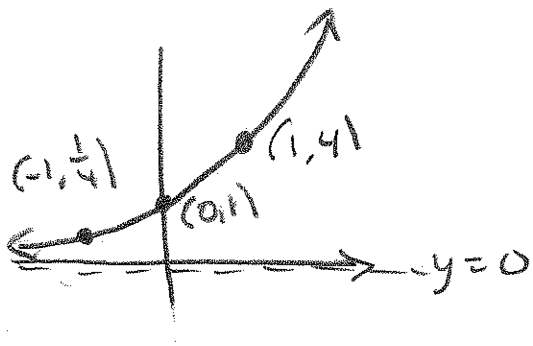
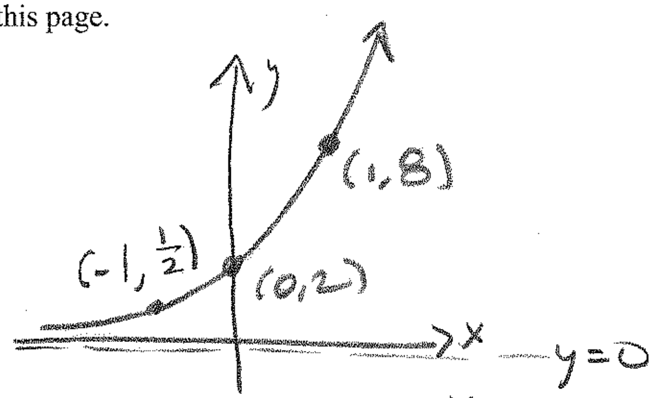
MAT 121 Test 4 - Fall, 2014 Name________________________________

70 Points Covers Chapter 4

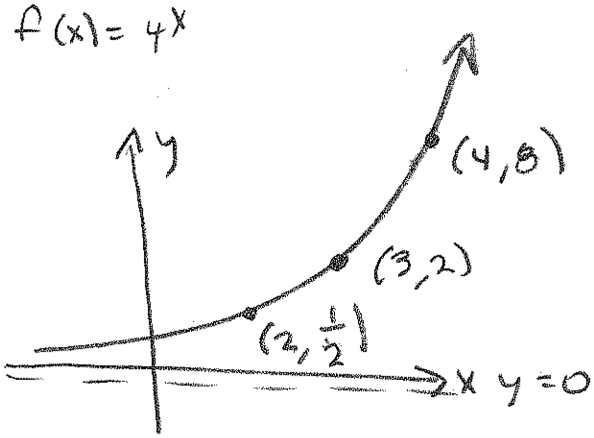
1. (15 pts) Starting with $f(x)=4^x$, sketch the graph of $g(x)=2\cdot 4^{x-3}-9$ in 4 steps (counting $f(x)=4^x$ as the first step). Use $x=-1$, $x=0$, and $x=1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Finding the x- and y-intercepts is a separate problem, so don't worry about them, on this page.



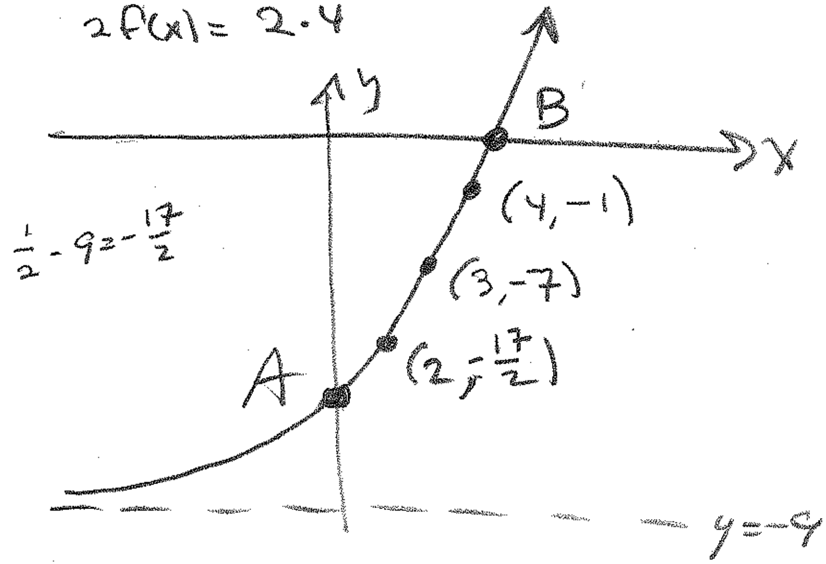
$f(x)=4^x$



$2f(x)=2\cdot 4^x$



$2f(x-3)=2\cdot 4^{x-3}$



$2\cdot f(x-3)-9=g(x)$

$=2\cdot 4^{x-3}-9$

A: $g(0)=2\cdot 4^{(0-3)}-9$

$=2\cdot 4^{-3}-9$

$=\frac{2}{64}-9$

$=\frac{1}{32}-\frac{288}{32}$

$=-\frac{287}{32}$

$\boxed{A=(0,-\frac{287}{32})}$

$\approx(0,-8.96875)$

B: $g(x)=0$

$2\cdot 4^{x-3}-9=0$

$2\cdot 4^{x-3}=9$

$4^{x-3}=\frac{9}{2}$

$x-3=\log_4(9/2)$

$x=\log_4(9/2)+3$

$\boxed{B=(\log_4(9/2)+3,0)\approx(4.0850,0)}$

2. Let $f(x)=\sqrt{2x+4}$ and $g(x)=\frac{x-2}{x-7}$.

a. (5 pts) What is the domain of f?

$2x+4\geq 0$

$2x\geq -4$

$\{x \mid x\geq -2\}=[-2,\infty)=\mathcal{D}$

b. (5 pts) What is the domain of g?

$x-7\neq 0$

$\{x \mid x\neq 7\}=(-\infty,7)\cup(7,\infty)=\mathcal{D}$

c. (5 pts) Write the function $\frac{f}{g}$. Do not simplify.

$$\frac{\sqrt{2x+4}}{\left(\frac{x-2}{x-7}\right)}$$

d. (5 pts) Write the function $f\circ g$. Do not simplify.

$$\sqrt{2\left(\frac{x-2}{x-7}\right)+4}$$

e. (5 pts) What is the domain of $\frac{f}{g}$?

$\frac{x-2}{x-7}=0$

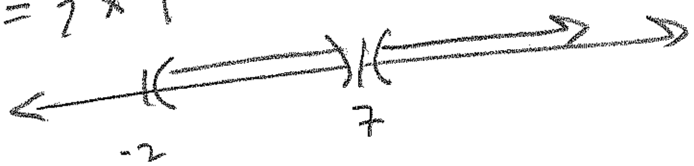
$x-2=0$

$x=2$

$\{x \mid x\geq -2 \text{ and } x\neq 7 \text{ and } \frac{x-2}{x-7}\neq 0\}$

$=\{x \mid x\geq -2 \text{ and } x\neq 7 \text{ and } x\neq 2\}$

$=\{x \mid x>-2 \text{ and } x\neq 7\}=\boxed{(-2,7)\cup(7,\infty)}$



f. (~~5~~ 10 pts) What is the domain of $f\circ g$?

$\frac{x-2}{x-7}\geq -2$

$\frac{x-2}{x-7}\geq \frac{-2(x-7)}{x-7}$

$\frac{x-2+2(x-7)}{x-7}\geq 0$

$\frac{x-2+2x-14}{x-7}\geq 0$

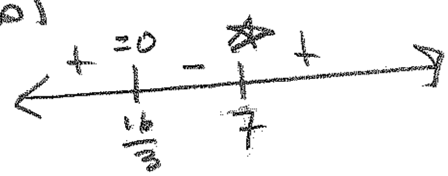
$\frac{3x-16}{x-7}\geq 0$

$\{x \mid x\in\mathcal{D}(g) \text{ and } g(x)\in\mathcal{D}(f)\}$

$=\{x \mid x\neq 7 \text{ and } \frac{x-2}{x-7}\geq -2\}$

$=\{x \mid x\neq 7 \text{ and } (x\leq \frac{16}{3} \text{ OR } x>7)\}$

$=(-\infty,\frac{16}{3}]\cup(7,\infty)$



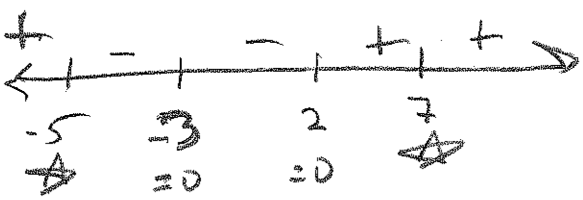
3. (5 pts) Let $g(x)=2\cdot 4^{x-3}-9$. Find the x- and y-intercepts for this function, rounded to 4 decimal places. For 5 **bonus** points, label these intercepts on your final graph on page 1.

See Pg 1

4. Find the domain:

a. (5 pts) $\sqrt{\dfrac{(x-2)(x+3)^2}{(x-7)^4(x+5)}}$

$$\frac{(x-2)(x+3)^2}{(x-7)^4(x+5)} \geq 0$$



$\mathcal{D} = (-\infty,-5)\cup\{-3\}\cup[2,7)\cup(7,\infty)$

b. (5 pts) $\log_3\left(\dfrac{(x-2)(x+3)^2}{(x-7)^4(x+5)}\right)$

$\mathcal{D} = (-\infty,-5)\cup(2,7)\cup(7,\infty)$

5. (5 pts) Solve $\log_7(x-4)+\log_7(x+2)=1$

$\log_7\big((x-4)(x+2)\big)=1$

$x^2-2x-8=7$

$x^2-2x-15=0$

$(x-5)(x+2)=0$

$x\in\{-2,5\}$, $-2\notin\mathcal{D}$

6. (5 pts) Solve $2^{x^2-8}\cdot 2^{-3x}=4$

$2^{x^2-3x-8}=2^2$

$x^2-3x-8=2$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x\in\{\cancel{-2},5\}$

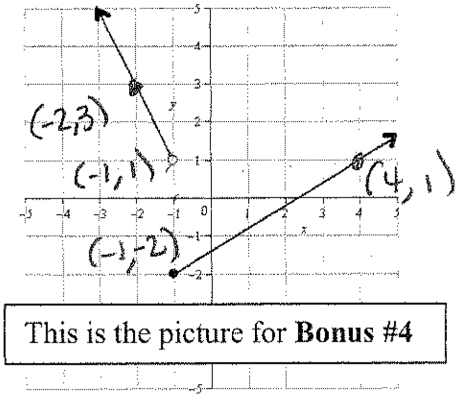
$\boxed{x\in\{5\}}$

Solve any two (2) Bonus problems for up to 10 points. I'll grade the first two I come to.

1. BONUS (5 pts) Solve the absolute value inequality $|2x-7| \geq 8$

2. BONUS (5 pts) Find the inverse function for $f(x) = \sqrt{2x-6}+1$. Then state the domain and range for both f and f^{-1}.

3. BONUS (5 pts) Re-write the function $g(x) = 5x^2 + 10x - 19$ in the form $g(x) = a(x-h)^2 + k$. State the vertex of this parabola.



This is the picture for **Bonus #4**

4. BONUS (5 pts) Write the formula for the piecewise-defined function shown, above right.

1. $|2x-7| \geq 8$

$2x-7 \geq 8$ OR $2x-7 \leq -8$

$2x \geq 15$ OR $2x \leq -1$

$\{x \mid x \geq \frac{15}{2}$ OR $x \leq -\frac{1}{2}\}$

$-\frac{1}{2}$ $\quad$ $\frac{15}{2}$

$(-\infty, -\frac{1}{2}] \cup [\frac{15}{2}, \infty)$

2. $\sqrt{2y-6}+1 = x$ $\qquad \mathcal{D}(f) = [3,\infty) = \mathcal{R}(f^{-1})$

$\sqrt{2y-6} = x-1$ $\qquad \mathcal{R}(f) = [1,\infty) = \mathcal{D}(f^{-1})$

$2y-6 = (x-1)^2$

$2y = (x-1)^2 + 6$

$y = \boxed{\frac{1}{2}(x-1)^2 + 3 = f^{-1}(x)}$

OR $\frac{1}{2}x^2 - x + \frac{7}{2}$

3. $5x^2 + 10x - 19$

$= 5(x+2x+1^2) - 5(1)^2 - 19$

$\boxed{= 5(x+1)^2 - 24 \quad (h,k) = (-1,-24)}$

ALTERNATE:

$\frac{-b}{2a} = -\frac{10}{2(5)} = -1 = h$

$f(-\frac{b}{2a}) = f(-1) = 5(-1)^2 + 10(-1) - 19$

$= 5 - 29 = -24 = k$

$\therefore f(x) = 5(x-(-1))^2 - 24 = 5(x+1)^2 - 24$

4. $m_1 = \frac{1-3}{-1+2} = \frac{-2}{1} = -2$

$y = -2(x-(-1)) + 1$

$= -2x - 2 + 1 = -2x - 1$

$m_2 = \frac{-2-1}{-1-4} = \frac{-3}{-5} = \frac{3}{5}$

$y = \frac{3}{5}(x-(-1)) - 2$

$\boxed{= \frac{3}{5}x - \frac{7}{5}}$

$$f(x) = \begin{cases} -2x-1 & \text{if } x < -1 \\ \frac{3}{5}x - \frac{7}{5} & \text{if } x \geq -1 \end{cases}$$