MAT 121
100 Points   Covers Chapter 2   Test 1 – Fall, 2014   Name KEY

NO GRAPHING CALCULATORS!!!

1. Consider the relation $f = \{(-2,3),(1,5),(2,3),(3,-2)\}$.

   a. (5 pts) Is f a function? Yes

   b. (5 pts) What is the domain of f? $\{-2, 1, 2, 3\}$

   c. (5 pts) What is the range of f? $\{3, 5, -2\}$

   d. (5 pts) Is f one-to-one? If not, explain why not. No. $(-2,3)$ and $(2,3)$ have same 2^{nd} Coord.

3. Let $f(x) = \sqrt{x-2}$ and $g(x) = \dfrac{x-7}{x+5}$.

   a. (5 pts) Write the function $\dfrac{f}{g}$. Do not simplify. $\dfrac{\sqrt{x-2}}{\frac{x-7}{x+5}}$

   b. (5 pts) What is the domain of $\dfrac{f}{g}$? $\{x \mid x \ge 2 \text{ and } x \ne -5 \text{ and } x \ne 7\}$ $\boxed{[2,7) \cup (7,\infty)}$

   $\mathcal{D}(g) = \{x \mid x \ne -5\}$

   $\mathcal{D}(f) = \{x \mid x \ge 2\}$

   $g(x) \ne 0$ $\quad x \ne 7$

   $\dfrac{x-7}{x+5} \ne 0$

   $x - 7 \ne 0$

   c. (5 pts) Write the function $f \circ g$. Do not simplify. $\boxed{\sqrt{\dfrac{x-7}{x+5} - 2}}$

   ~~d. (5 pts) What is the domain of $f \circ g$?~~

$\boxed{\begin{aligned}&\{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}\\ &= \{x \mid x \ne -5 \text{ and } \tfrac{x-7}{x+5} \ge 2\}\end{aligned}}$ 5 pts $= \{x \mid x \ne -5$ and $\boxed{\text{Bonus } -17 \le x < -5}\}$

$\dfrac{x-7}{x+5} \ge \dfrac{2(x+5)}{x+5}$ Bonus

$\dfrac{x-7-2x-10}{x+5} = \dfrac{-x-17}{x+5} \ge 0$

No $-$ Yes Yes $+$ No No $-$

-17 $=0$ $\quad -5$

$= [-17, -5)$

4. (5 pts) Simplify the difference quotient for $f(x)=2x^2-3x$.

$$\frac{f(x+h)-f(x)}{h}=\frac{2(x+h)^2-3(x+h)-[2x^2-3x]}{h}$$

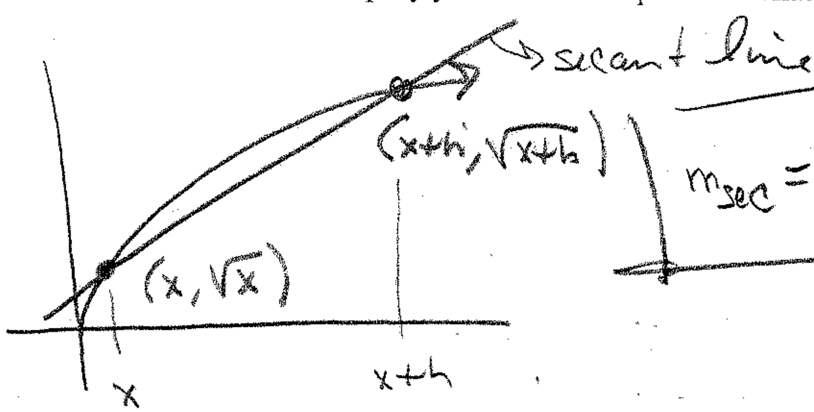
$$=\frac{2(x^2+2xh+h^2)-3x-3h-2x^2+3x}{h}=\frac{2x^2+4xh+2h^2-3h-2x^2}{h}$$

$$=\frac{4xh+2h^2-3h}{h}=\frac{h(4x+2h-3)}{h}=\boxed{4x+2h-3}$$

Bonus (5 pts) Pass to the limit as h approaches zero, and show me some calculus to go with #4.

$$\xrightarrow{h\to 0} 4x-3$$

5. (5 pts) Draw a picture for the difference quotient for $f(x)=\sqrt{x}$. Describe what the difference quotient represents, in words. Do not simplify your difference quotient. That's a bonus problem, later on.



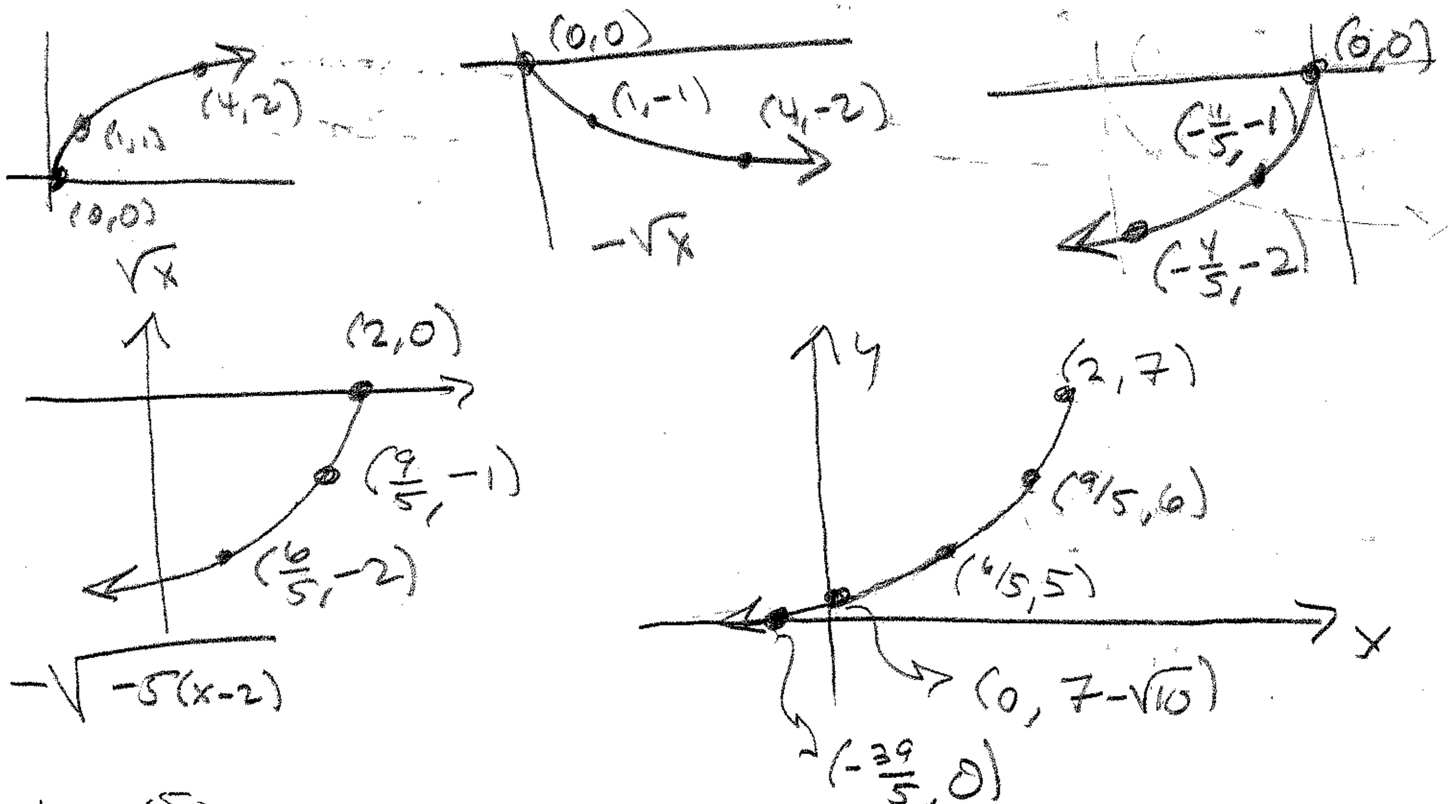
$$\boxed{m_{sec}=\frac{\sqrt{x+h}-\sqrt{x}}{h}}$$

It's the slope of a line connecting 2 points on the graph of $f(x)=\sqrt{x}$

6. Let $g(x) = -\sqrt{10-5x} + 7$. $\quad 10-5x = -5(x-2)$

a. (10 pts) Sketch the graph of $g(x)$, by transforming the basic function $f(x) = \sqrt{x}$. I want to see 3 points labeled in the graph of g – preferably starting with $(0,0)$, $(1,1)$ and $(4,2)$ – and track where those points are moved to after every step, as demonstrated in class.

$\sqrt{x} \quad -\sqrt{x} \quad -\sqrt{-5x} \quad -\sqrt{-5(x-2)} \quad -\sqrt{-5(x-2)} + 7$



$-\frac{1}{5} + 2\left(\frac{5}{5}\right) = \frac{-1+10}{5} = \frac{9}{5} \qquad \frac{-4+10}{5} = \frac{6}{5}$

b. (5 pts) State the domain and range of $g(x)$, based on your final graph.

$D = (-\infty, 2]$, $R = (-\infty, 7]$

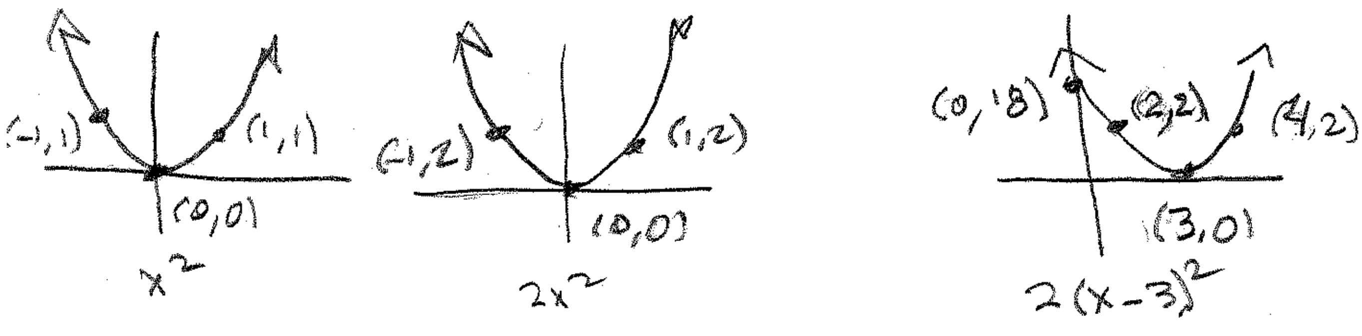
c. (5 pts) Find the x- and y-intercept of $g(x)$, and label them, clearly, on the graph.

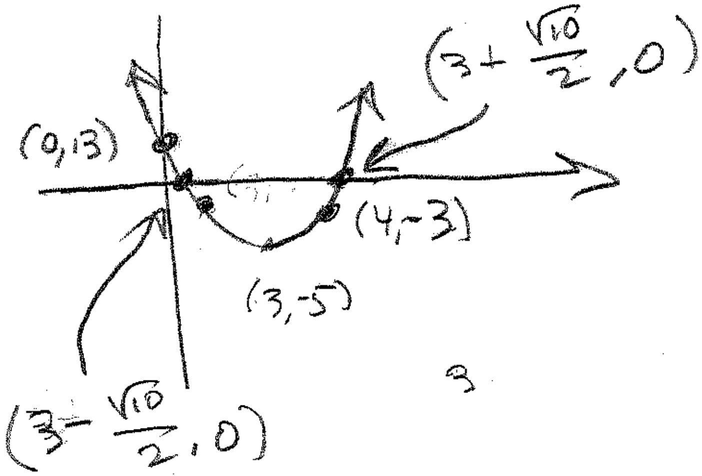
$g(0) = -\sqrt{10} + 7$

$$-\sqrt{10-5x} = -7$$
$$10-5x = 49$$
$$-5x = 39$$
$$x = -\frac{34}{5}$$

7. (10 pts) Sketch the graph of $r(x) = 2(x-3)^2 - 5$ by transforming the basic function $f(x) = x^2$. I want to see 3 points labeled in the graph of f, and I want you to track where those points are moved to after every step, as demonstrated in class.

$x^2 \quad 2x^2 \quad 2(x-3)^2 \quad 2(x-3)^2-5$





8. (5 pts) Find the x- and y-intercepts and add them to your final sketch, above. For x-intercept, leave final answer in simplified radical form.

$2(x-3)^2-5=0$

$2(x-3)^2=5$

$(x-3)^2=\frac{5}{2}$

$x-3=\pm\sqrt{\frac{5}{2}}=\pm\frac{\sqrt{5}\sqrt{2}}{\sqrt{2}\sqrt{2}}=\frac{\sqrt{10}}{2}$

$x=3\pm\frac{\sqrt{10}}{2}$ OR $\frac{6\pm\sqrt{10}}{2}$

9. (5 pts) Prove that $\frac{x+1}{x-3}$ is one-to-one.

$$\frac{x_1+1}{x_1-3} = \frac{x_2+1}{x_2-3}$$

$$\Rightarrow (x_1+1)(x_2-3) = (x_2+1)(x_1-3)$$

$$\Rightarrow x_1x_2 - 3x_1 + x_2 - 3 = x_2x_1 - 3x_2 + x_1 - 3$$

$$-3x_1 + x_2 = -3x_2 + x_1$$

$$-4x_1 = -4x_2 \Rightarrow x_1 = x_2 \blacksquare$$

10. (5 pts) Suppose y is jointly proportional to the square of x and the cube of z, and inversely proportional to u and the square root of w. Write an equation for this relationship between y, x, z, u, and w.

$$\boxed{y = k\frac{x^2z^3}{u\sqrt{w}}}$$

11. (5 pts) Explain why $x^2 + y^2 = 81$ does *not* define y as a function of x.

$$y^2 = 81 - x^2$$

$$y = \pm\sqrt{81-x^2}$$

$x=0 \Rightarrow y = \pm 9$ and we need ONE y-value, here

Answer two of the following for **Bonus** (5 pts each)

B1: Simplify the difference quotient for the function $f(x)=\sqrt{2x}$. Then pass to the limit, as h approaches zero.

B2: Complete the square to re-write the function $h(x)=5x^2-3x+2$ in the form $a(x-h)^2+k$. What is the vertex?

B3: What is the domain of $r(x)=\dfrac{x-5}{x^2-5x+6}$?

B4: What is the domain of $w(x)=\dfrac{x^{77}-5x^{12}+17x}{\sqrt{5-10x}}$

B5: Prove that $g(x)=-\sqrt{10-5x}+7$ is 1-to-1.

B6: Given $g(x)=-\sqrt{10-5x}+7$, find what $g^{-1}(x)$ is. $\left(\text{Hint}: (-x+7)^2=(x-7)^2\right)$

B7: Given $g(x)=-\sqrt{10-5x}+7$, find the domain and range of $g^{-1}(x)$.

B1

$$\frac{\sqrt{2(x+h)}-\sqrt{2x}}{h} = \left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{h}\right)\left(\frac{\sqrt{2(x+h)}+\sqrt{2x}}{\sqrt{2(x+h)}+\sqrt{2x}}\right)$$

$$= \frac{2x+h-2x}{h\left(\sqrt{2(x+h)}+\sqrt{2x}\right)} = \frac{h}{h\left(\sqrt{2(x+h)}+\sqrt{2x}\right)}$$

$$= \boxed{\frac{1}{\sqrt{2(x+h)}+\sqrt{2x}}}$$

B5

$$-\sqrt{10-5x_1}+7 = -\sqrt{10-5x_2}+7$$

$$-\sqrt{10-5x_1} = -\sqrt{10-5x_2}$$

$$\sqrt{10-5x_1} = \sqrt{10-5x_2}$$

$$10-5x_1 = 10-5x_2$$

$$-5x_1 = -5x_2$$

$$x_1 = x_2 \quad \blacksquare$$

B6

$$-\sqrt{10-5y}+7 = x$$

$$-\sqrt{10-5y} = x-7$$

$$\sqrt{10-5y} = -x+7$$

$$10-5y = (-x+7)^2 = (x-7)^2$$

$$-5y = (x-7)^2-10$$

$$y = \frac{(x-7)^2-10}{-5} = \frac{10-(x-7)^2}{5} = \boxed{2-\frac{1}{5}(x-7)^2 = g^{-1}(x)}$$

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B7 $-\sqrt{10-5x}+7=g(x)$

$10-5x \geq 0$

$-5x \geq -10$

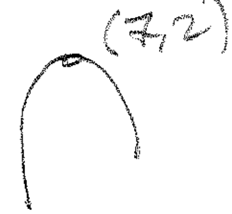
$x \leq 2$

$\mathcal{D}(g)=\{x \mid 10-5x \geq 0\}$

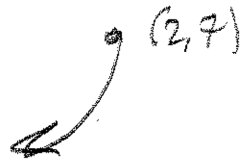
$=\{x \mid x \leq 2\}$

$g^{-1}(x) = -\frac{1}{5}(x-7)^2+2$, $(h,k)=(7,2)$

Not much help.



But $g(x)$ looks like



so $\mathcal{D}(g)=(-\infty,2]$

and $\mathcal{R}(g)=(-\infty,7]$

So $\boxed{\mathcal{D}(g^{-1})=(-\infty,7]}$

and $\boxed{\mathcal{R}(g^{-1})=(-\infty,2]}$