MAT 121 FINAL TEST, Fall, 2013

200 Points Name KEY

1. Solve the equation $x^2 + 4x - 21 = 0$ in two different ways:

part a (10 pts) Factoring

$(x+7)(x-3)=0$

$x \in \{-7, 3\}$

part b (10 pts) Completing the square

$x^2+4x = 21$

$x^2+4x+2^2 = 21+4$

$(x+2)^2 = 25$

$\sqrt{(x+2)^2} = \sqrt{25}$

$|x+2| = 5$

$x+2 = \pm 5$

$x = -2 \pm 5 \to 3, -7$

$x \in \{-7, 3\}$

2. Solve the absolute value inequality. Give your answer in set-builder *and* interval notation:

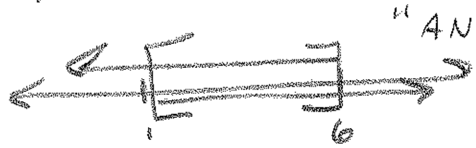
part a (10 pts) $|2x-7| \le 5$

$2x-7 \le 5$ and $2x-7 \ge -5$

$2x \le 12$ $\quad$ $2x \ge 2$

$\{x \mid x \le 6$ and $x \ge 1\}$

"AND"

$=$ 

$= [1, 6]$

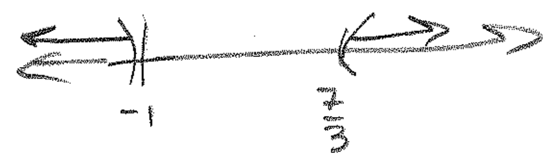
part b (10 pts) $|3x-2| > 5$

$3x-2 > 5$ OR $3x-2 < -5$

$3x > 7$ $\quad$ $3x < -3$

$\{x \mid x > \frac{7}{3}$ OR $x < -1\}$

"OR"

$=$ 

$= \boxed{(-\infty, -1) \cup (\frac{7}{3}, \infty)}$

part c (5 pts) $|7x+2| \ge -4$

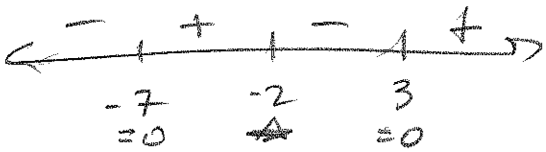
$\mathbb{R}$

part d (5 pts) $|2x-7| < -4$

$\varnothing$

3. (10 pts) What is the domain of $f(x)=\sqrt{\dfrac{x^2+4x-21}{x+2}}$?

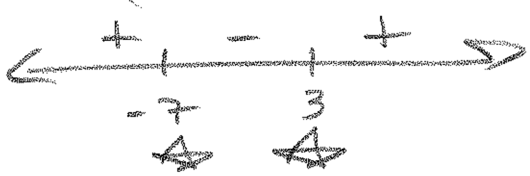
$$\frac{(x+7)(x-3)}{x+2} \geq 0$$



$$\boxed{x \in [-7,-2) \cup [3,\infty) = \mathcal{D}}$$

4. (10 pts) What is the domain of $\log_5\left(x^2+4x-21\right)$?

$$(x+7)(x-3) > 0$$



$$\boxed{\mathcal{D} = (-\infty,-7) \cup (3,\infty)}$$

5. Let $f(x)=4x^4-16x^3-31x^2+94x-195$.

part a (10 pts) Use synthetic division to determine if $x+3$ is a factor of f.

Interpret the your work by filling in the *quotient* and *remainder* in the statement $4x^4-16x^3-31x^2+94x-195=(x+3)\cdot\textit{quotient}+\textit{remainder}$.

Hopefully, your remainder is zero. It's how you get it and how you interpret it that matter to me.

$$\begin{array}{r|rrrrr} -3 & 4 & -16 & -31 & 94 & -195 \\ & & -12 & 84 & -159 & 195 \\ \hline & 4 & -28 & 53 & -65 & 0 \end{array}$$

$(x+3)(4x^3-28x^2+53x-65)+0$

part b (10 pts) Show that $x=5$ is a root of f by dividing your *quotient* in **part a** by $x-5$. The quotient from **part a** is the so-called *depressed polynomial*, of degree 3. This question, in itself, ought to give you a very clear idea of what your conclusion ought to have been in part a.

$$\begin{array}{r|rrrr} 5 & 4 & -28 & 53 & -65 \\ & & 20 & -40 & 65 \\ \hline & 4 & -8 & 13 & 0 \end{array}$$

part c (10 pts) Compute the discriminant of $4x^2-8x+13$. Then find the two nonreal roots of $4x^2-8x+13$, by any method (short of copying from someone else). This question *should* give you a very good idea of how things went for you, above.

$a=4,\ b=-8,\ c=13$

$b^2-4ac=(-8)^2-4(4)(13)$

$=64-208$

$=-144$

$\sqrt{-144}=12i$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{8\pm 12i}{2(4)}=\dfrac{4(2\pm 3i)}{2(4)}$

$=\dfrac{2\pm 3i}{2}$ OR $1\pm\frac{3}{2}i$

part d (10 pts) Write f as the product of linear factors. You can still earn the **part d** points without any of **a**, **b**, or **c** by *making up* plausible answers and incorporating them into the answer to this question. It should have 2 real and 2 nonreal zeros represented by the factors.

$4(x+3)(x-5)(x-(1+\frac{3}{2}i))(x-(1-\frac{3}{2}i))$

$$\begin{array}{r} 1536 \\ 2 \\ \hline 3072 \end{array}$$

6. (10 pts) Determine a, r and n for the finite geometric series $3+6+12+....+1536$

Use a, r, and n to determine the sum by the formula $\sum_{k=1}^{n} a \cdot r^{k-1} = a\left(\frac{r^n - 1}{r - 1}\right)$.

$\boxed{a=3,\ r=\frac{6}{3}=\frac{12}{6}=2}$

$3 \cdot 2^9 = 3 \cdot 2^{n-1}$

$9 = n-1$

$\boxed{10 = n}$

$S_{10} = 3\left(\frac{2^{10}-1}{2-1}\right) = 3\left(\frac{1024-1}{1}\right)$

$= 3(1023) = \boxed{3069}$

$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
$2^{10} = 1024$

2 | 1536
2 | 768
2 | 384
2 | 192
2 | 96
2 | 48
2 | 24
2 | 12
2 | 6
3

7. (10 pts) Use Pascal's Triangle (Binomial Theorem) to expand the binomial power $(x-2)^6$.

Expanding without using a recognizable version of this technique will earn at most 2 points.

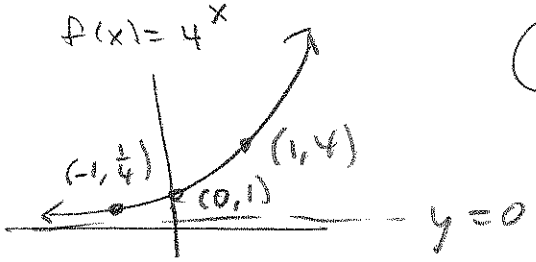
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$x^6(-2)^0 + 6x^5(-2)^1 + 15x^4(-2)^2 + 20x^3(-2)^3 + 15x^2(-2)^4 + 6x^1(-2)^5 + x^0(-2)^6$

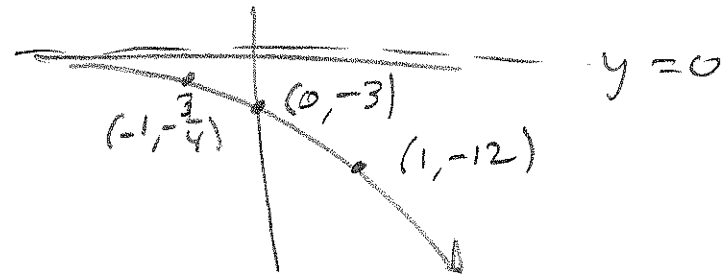
$= \boxed{x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x^1 + 64}$

8. (10 pts) Graph $g(x) = -3 \cdot 4^{x-2} + 6$ using the techniques of shifting and reflecting as done in class. Start with the graph of the basic function $f(x) = 4^x$ and show all stages. In the final graph, indicate (label as ordered pairs) the x- and y- intercepts.

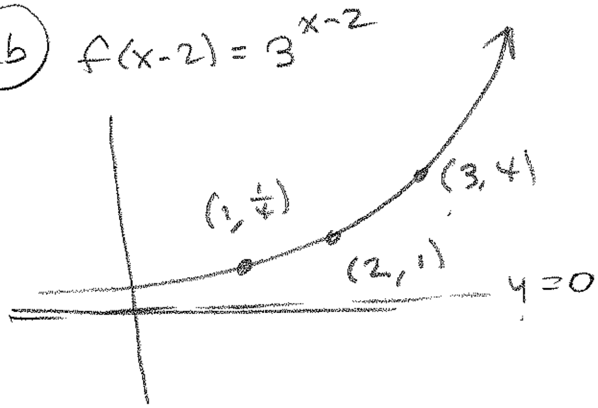
$f(x) = 4^x$



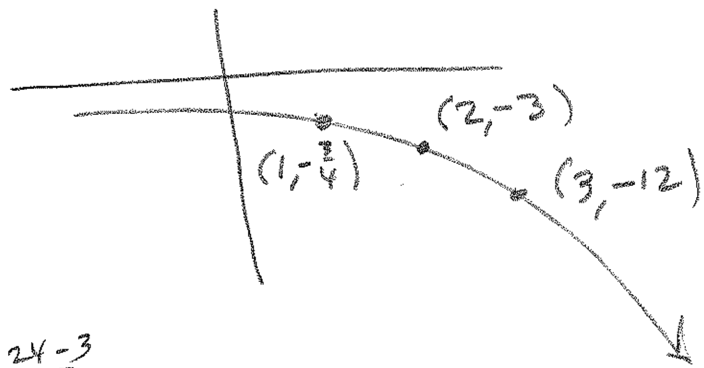
2a $-3f(x) = -3 \cdot 4^x$



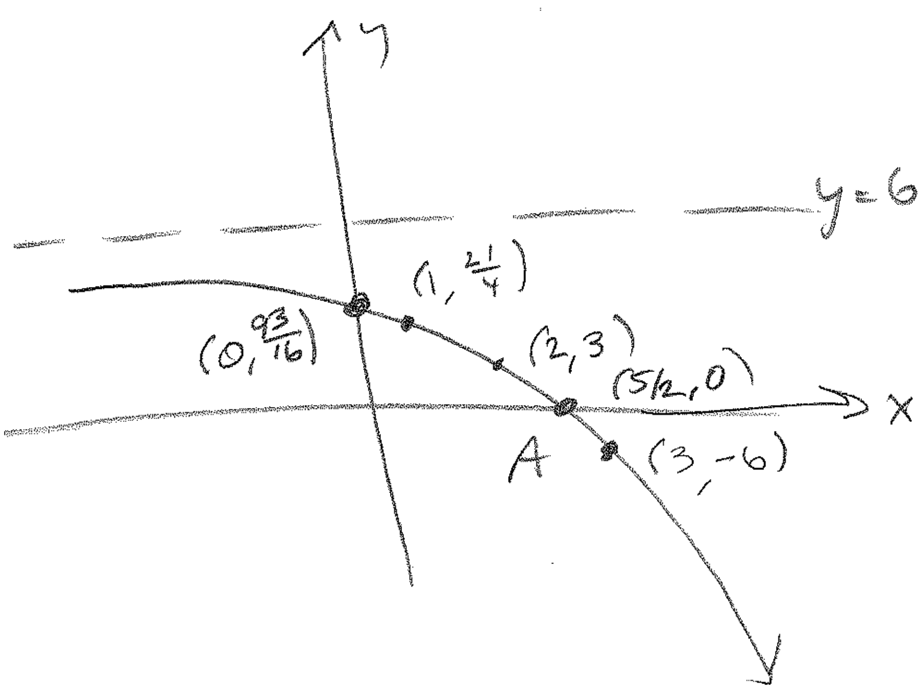
2b $f(x-2) = 3^{x-2}$



3 $-3f(x-2) = -3 \cdot 4^{x-2}$



4 $-3f(x-2) + 6 = g(x) =$
$= -3 \cdot 4^{x-2} + 6$ $\qquad 6 - \frac{3}{4} = \frac{24-3}{4}$



$g(0) = 3 \cdot 4^{-2} + 6$
$= \frac{-3}{16} + \frac{96}{16} = \frac{93}{16}$

$-3 \cdot 4^{x-2} + 6 = 0$
$4^{x-2} = 2$
$x - 2 = \log_4(2)$
$x = 2 + \log_4(2)$
$= 2 + \log_4(4^{\frac{1}{2}})$
$= 2 + \frac{1}{2}$
$= \frac{5}{2}$

$\boxed{A = (\frac{5}{2}, 0)}$

9. (15 pts) Solve the equation $A_0 e^{5300k} = \frac{1}{2} A_0$ for the decay rate, k.

$e^{5300K} = \frac{1}{2}$

$\ln(e^{5300K}) = \ln(1/2) = -\ln 2$

$5300K = -\ln 2$

$K = -\frac{\ln 2}{5300}$

10. (10 pts) Suppose the half-life of Carbon-14 is 5300 years. (It isn't, but just suppose…). How old is a sample of wood from a fire pit if only 30% of the original amount of Carbon-14 remains?

$A_0 e^{Kt} = .35 A_0$ Solve for t (K is done)

$e^{Kt} = .35$

$Kt = \ln(.35)$

$t = \frac{\ln(.35)}{K} = \frac{\ln(.35)}{\frac{-\ln 2}{5300}} = \frac{5300 \ln(.35)}{-\ln(2)}$

$\approx$

11. (15 pts) Solve the system of linear equations:

$3x + 2y = 6$

$x - 3y = 12$

$x = 3y + 12$

$3(3y + 12) + 2y = 6$

$9y + 36 + 2y = 6$

$11y = -30$

$\boxed{y = -\frac{30}{11}}$

$x = 3\left(-\frac{30}{11}\right) + 12$

$= -\frac{90}{11} + \frac{132}{11}$

$= \boxed{\frac{42}{11} = x}$

$(x, y) = \left(\frac{42}{11}, -\frac{32}{11}\right)$

BONUS

1) The population of a bee colony in 2008 is 800 bees. The population of that colony grows to 900 in 2012. The population is a function of time in the exponential model $P(t) = P_0 e^{kt}$ where $t = 0$ represents the year 2008.

a) Define the variables given this information and identify the two ordered pairs to use as points.

t = time in yrs after 2008

P = P(t) = Pop as func. of ~~yrs after 2008~~ t.

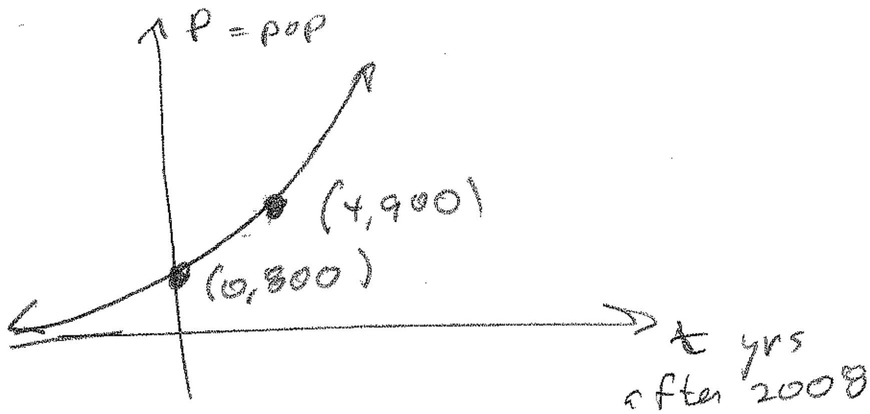
GIVEN: (0, 800) & (4, 900)

2012 − 2008 = 4

b) Graphing

i) Label the axes appropriately for the context of the problem.

ii) Graph (plot) the 2 points. (*Just* the two points, in correct (relative) position. We will finish the graph later.)



c)

i) Find the growth rate. Show your work. Round to 4 decimal places.

ii) Find the equation of the exponential function which models the situation.

(ii) $\boxed{P(t) \approx 800e^{.0294t}}$

$P(t) = 800e^{kt}$

$P(4) = 800e^{4k} = 900$

$e^{4k} = \frac{9}{8}$

$4k = \ln(9/8)$

$k = \frac{1}{4}\ln(9/8) \approx .02944758$

(i) $\boxed{k \approx .0294}$

d) Graph the equation of the curve on the same graph as the two points in part b. (I'd *rather* you just did the graph of the thing (correct *shape*) and then stuck the two points on it in relatively correct position.)

DONE

e) Use your equation (with *k* rounded to 4 places) to find the estimated population in 2017. Show your work.

$2017 - 2008 = 9$

Want $P(9) = 800e^{9k} \approx 800e^{9(.0294)}$

$\approx 1042.327766 \approx \boxed{1042 \text{ Bees}}$

f) Use the equation to calculate in what year the population will reach 1000 if the growth continues at this same rate. Show your work.

Solve $P(t) = 800e^{kt} = 1000$ for t

$e^{kt} = \frac{1000}{800} = \frac{5}{4}$

$kt = \ln\left(\frac{5}{4}\right)$

$t = \frac{\ln\left(\frac{5}{4}\right)}{k} \approx \frac{\ln\left(\frac{5}{4}\right)}{.0294} \approx \boxed{7.58991671 \approx 8 \text{ yrs}}$

g) What would be the effect to the population if the rate had the opposite sign? Use complete sentences in your explanation.

It would be exponential decay!

h) List two real-life factors which may affect the population such that this model would not prove valid. Use complete sentences.

Changes in climate, habit, and ~~predation~~, ~~vegetat~~ are among the factors that can affect a simple pop. model such as this.