MAT 121 f2f, FINAL TEST, Fall, 2011
200 Points

Name KEY

This is our final learning opportunity together, and I'm hoping to take full advantage. Read the questions carefully. Sometimes, you can earn points on a problem by *knowing* that you did it wrong and *explaining* how you know and what you're *trying* to accomplish, and *how* you're going about it.

1. Solve the equation $x^2 - 2x - 15 = 0$ in three different ways:

part a (10 pts) Factoring

$(x-5)(x+3)=0$

$x \in \{-3, 5\}$

part b (15 pts) Completing the square

$x^2 - 2x = 15$

$x^2 - 2x + 1^2 = 16$

$(x-1)^2 = 16$

$x - 1 = \pm 4$

$x = 1 \pm 4 \to 5, -3$

$x \in \{-3, 5\}$

part c (15 pts) Quadratic formula

$a = 1, b = -2, c = -15$

$b^2 - 4ac = (-2)^2 - 4(1)(-15)$

$= 4 + 60 = 64$

$x = \frac{-(-2) \pm \sqrt{64}}{2(1)} = \frac{2 \pm 8}{2} \to \frac{10}{2} = 5, \; -\frac{6}{2} = -3$

$x \in \{-3, 5\}$

2. Solve the absolute value inequality. Give your answer in set-builder *and* interval notation.

part a (10 pts) $|7x+2| \geq 4$

$7x+2 \geq 4$ OR $7x+2 \leq -4$

$7x \geq 2$ $\quad$ $7x \leq -6$

$\{x \mid x \geq \frac{2}{7}$ OR $x \leq -\frac{6}{7}\}$

$(-\infty, -\frac{6}{7}] \cup [\frac{2}{7}, \infty)$

part b (10 pts) $|2x-7| < 4$

$-4 < 2x-7 < 4$

$3 < 2x < 11$

$\{x \mid \frac{3}{2} < x < \frac{11}{2}\}$

$x \in (\frac{3}{2}, \frac{11}{2})$

3. Let $f(x) = \sqrt{x-14}$ and $g(x) = x^2 - 3x - 14$

part a (15 pts) What's the domain of $f(x)$? Give the answer in set-builder and interval notation.

$x - 14 \geq 0$

$\{x \mid x \geq 14\} = [14, \infty)$

part b (15 pts) Determine $(f \circ g)(x)$. Simplify your answer.

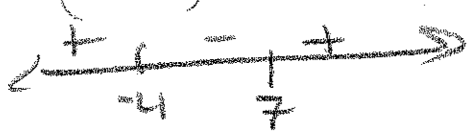
$\sqrt{x^2 - 3x - 14 - 14} = \sqrt{x^2 - 3x - 28}$

part c (5 pts) What's the domain of $(f \circ g)(x)$? Give your answer in set-builder and interval notation.

NEED $x^2 - 3x - 14 \geq 14$ $\Longrightarrow$

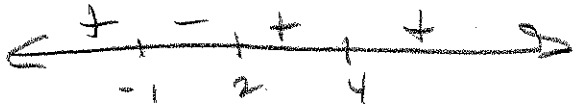
$x^2 - 3x - 28 \geq 0$

$(x-7)(x+4) \geq 0$

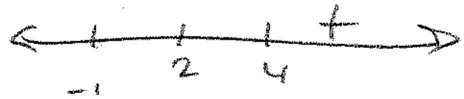


$\boxed{(-\infty, -4] \cup [7, \infty) = \{x \mid x \leq -4 \text{ OR } x \geq 7\}}$

4. (15 pts) Solve $(x-2)^3(x+1)(x-4)^2 > 0$. Give the solution set in interval notation.



$x \in (-\infty,-1) \cup (2,4) \cup (4,\infty)$

I got this: 

just by looking. The rest's managing sign changes by observing even/odd powers.

5. (10 pts) What is the domain of $h(x) = \sqrt{(x-2)^3(x+1)(x-4)^2}$?

$x \in (-\infty,-1] \cup [2,4] \cup [4,\infty)$

$= \boxed{(-\infty,-1] \cup [2,\infty)}$

Same as previous, only $\geq$ instead of $>$.

6. (10 pts) What is the domain of $\sqrt{\dfrac{(x-2)^3(x+1)}{(x-4)^2}}$? The hard part's done...

$x \neq 4$, but otherwise same as #5:

$x \in (-\infty,-1] \cup [2,4) \cup (4,\infty)$

Same as previous (#5) only $x=4$ is thrown out!

7. (10 pts) Use synthetic division to find $f(3)$ for $f(x) = x^4 - 5x^3 - 3x^2 + 43x - 60$

$$\begin{array}{r|rrrrr} 3 & 1 & -5 & -3 & 43 & -60 \\ & & 3 & -6 & -27 & 48 \\ \hline & 1 & -2 & -9 & 16 & \boxed{-12 = f(3)} \end{array}$$

8. (10 pts) Determine a, r and n for the finite geometric sequence $2, \frac{4}{5}, \frac{8}{25}, \ldots, \frac{256}{78125}$

sum – make it a sum.

Should be a sum

Use a, r, and n to determine the sum by the formula $\sum_{k=1}^{n} a \cdot r^{k-1} = a\left(\frac{1-r^n}{1-r}\right)$. A fractional answer is better, but I'll give you most of the points if you provide a decimal answer that is accurate to 4 decimal places.

$$\frac{256}{78125} = \frac{2^8}{5^7} = 2 \cdot \frac{2^7}{5^7}$$

$$2 \cdot \frac{2^7}{5^7} = 2 \cdot \frac{2^{n-1}}{5^{n-1}} \Rightarrow$$

$$n = 8, \; a = 2, \; r = \frac{2}{5}$$

$5 \overline{)78125}$, $5 \overline{)15625}$, $5 \overline{)3125}$, $5 \overline{)625}$, $5 \overline{)125}$, $5 \overline{)25}$, 5

1 $2 \overline{)256}$, 2 $2 \overline{)128}$, 3 $2 \overline{)64}$, 4 $2 \overline{)32}$, 5 $2 \overline{)16}$, 6 $2 \overline{)8}$, 7 $2 \overline{)4}$, 8 2

$$78125 \times 5 = 390625$$

$$\Rightarrow S = 2\left(\frac{1-\left(\frac{2}{5}\right)^8}{1-\frac{2}{5}}\right)$$

$$= \frac{2\left(\frac{390625-256}{390625}\right)}{\frac{3}{5}} = \frac{5}{3}(2)\left(\frac{390369}{390625}\right) = 2\left(\frac{130123}{78125}\right) = \boxed{\frac{260246}{78125}}$$

≈ 3.33114

3.3311488

9. (10 pts) Find the sum of the infinite series $\sum_{k=1}^{\infty} 4 \cdot \left(\frac{3}{4}\right)^{k-1} = 4 + 4 \cdot \frac{3}{4} + 4 \cdot \left(\frac{3}{4}\right)^2 + \ldots\ldots$

$$4\left(\frac{1}{1-\frac{3}{4}}\right) = 4\left(\frac{1}{\frac{1}{4}}\right) = 4 \cdot 4 = \boxed{16}$$

Finance Formulas:

$$A = P\left(1+\frac{r}{m}\right)^{mt} = P(1+r)^n \qquad FV = R\left(\frac{(1+i)^n - 1}{i}\right)$$

10. (10 pts) What's the future value, in 10 years, of \$10,000 deposited into a savings account, earning 4.3% annual percentage rate compounded daily?

$$A = 10000\left(1+\frac{.043}{365}\right)^{(365)(10)} \approx \boxed{\$15372.19}$$

11. (10 pts) An annnuity consists of monthly payments of \$600 into an account earning 8.4% annual interest, compounded monthly, for 10 years. There are two ways to ask this question:

First way: How much does JG Wentworth feel that this annuity is worth?

Second way: If the annuity described is actually your monthly loan payments, how much did you borrow in the first place?

$$A = FV$$

$$P(1+i)^n = R\left(\frac{(1+i)^n - 1}{i}\right)$$

$$\boxed{\text{Check: } \$72000 \text{ total paid}}$$

$$2.309598381\,P = 112251.2898$$

$$P = R\left(\frac{1-(1+i)^{-n}}{i}\right)$$

$$P \approx \frac{112251.2898}{2.309598381} \approx 48602.08$$

$$= 600\left(\frac{1-\left(1+\frac{.084}{12}\right)^{-(12)(10)}}{\frac{.084}{12}}\right) \approx \boxed{\$48,602.08}$$

Bonus (10 pts) Use Pascal's Triangle (Binomial Theorem!) to help you expand $(x-2)^4$. Expanding without using this technique will not earn any points.

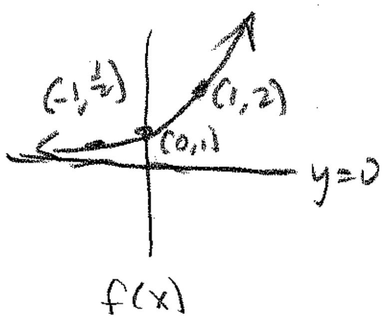
$$x^4 + 4(x)^3(-2)^1 + 6(x)^2(-2)^2 + 4(x)(-2)^3 + (-2)^4$$

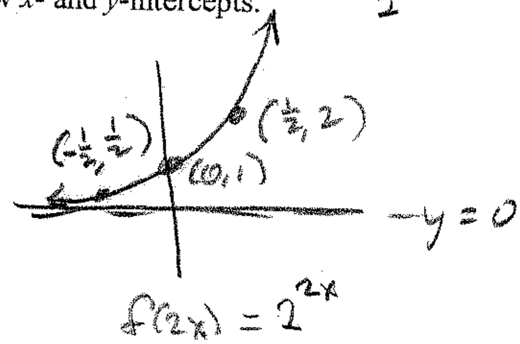
$$= x^4 - 8x^3 + 24x^2 - 32x + 16$$

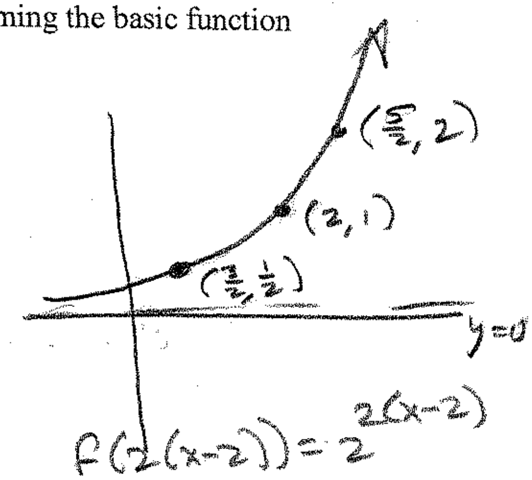
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 ~~10~~ 5 1

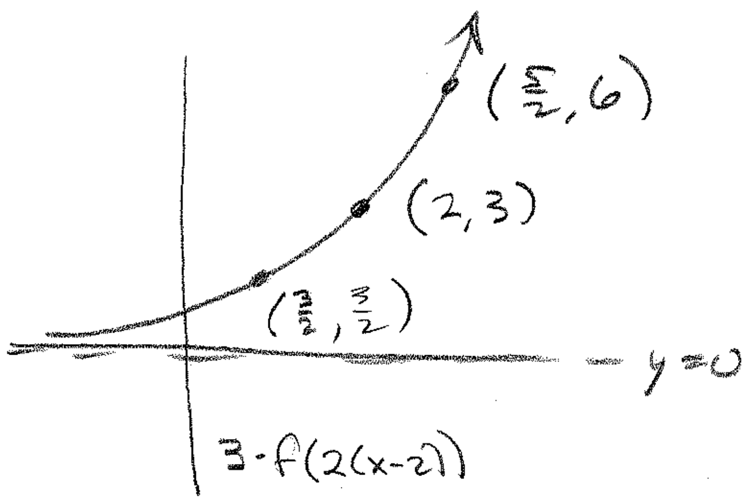
12. (15 pts) Sketch the graph of $g(x) = 3 \cdot 2^{2x-4} - 5$ by transforming the basic function $f(x) = 2^x$. Show x- and y-intercepts.

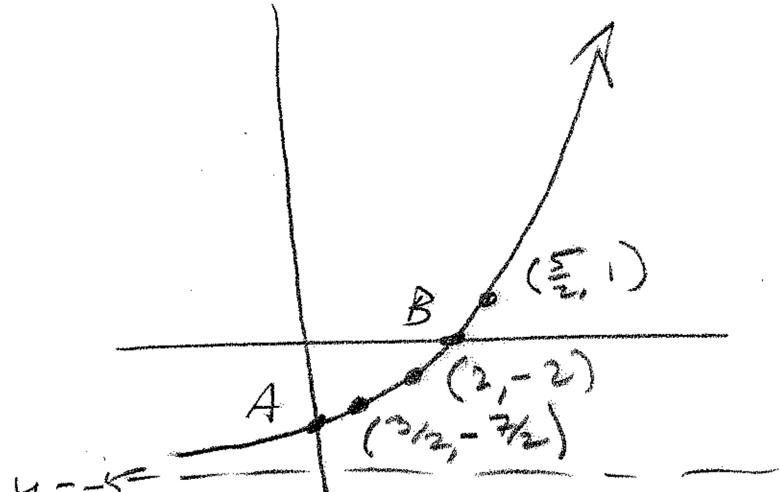
$2^{2(x-2)}$











$$\boxed{A = (0, f(0)) = \left(0, -\frac{73}{5}\right)}$$

$$3 \cdot 2^{2(0)-4} - 5$$
$$= 3 \cdot 2^{-4} - 5$$
$$= \frac{3}{16} - 5$$
$$= \frac{3-80}{16}$$
$$= -\frac{77}{16} = 4.8125$$

B: $3 \cdot 2^{2x-4} - 5 = 0$
$$3 \cdot 2^{2x-4} = 5$$
$$2^{2x-4} = \frac{5}{3}$$
$$2x - 4 = \log_2\left(\frac{5}{3}\right)$$
$$2x = 4 + \log_2\left(\frac{5}{3}\right)$$
$$x = \frac{4 + \log_2\left(\frac{5}{3}\right)}{2}$$
$$\boxed{B = \left(\frac{4 + \log_2\left(\frac{5}{3}\right)}{2}, 0\right)}$$
$$\approx (2.3685, 0)$$